Fundamentals of Geodesy

Earth Coordinate system, Geoid, Ellipsoid, MSL &Map Projections

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Mother Earth ?

How big is the Earth?

shape of the planet ?

How tall is a mountain?

Where my property ends?

Where am I?

How far am I from a place?

In which direction should I go?

How big is my property?

Geodesy answers these questions

Geodesy, Map Projections and Coordinate Systems

- Geodesy - the shape of the earth and definition of earth datum, gravity field, earth rotation.

- Map Projection - Transformation of the curved earth to a flat map.(3D->2D).

- Coordinate systems -(x,y,z) coordinate systems for map data.

Why care about the shape of the Earth?

- **If we want to have a mathematical representation of a point (point's coordinate) we need to have a reference surface we can refer to.**
- **Knowing the shape of the Earth we can define this surface.**
- **The gravity field gives the best representation(physical model) of the shape of the Earth.**

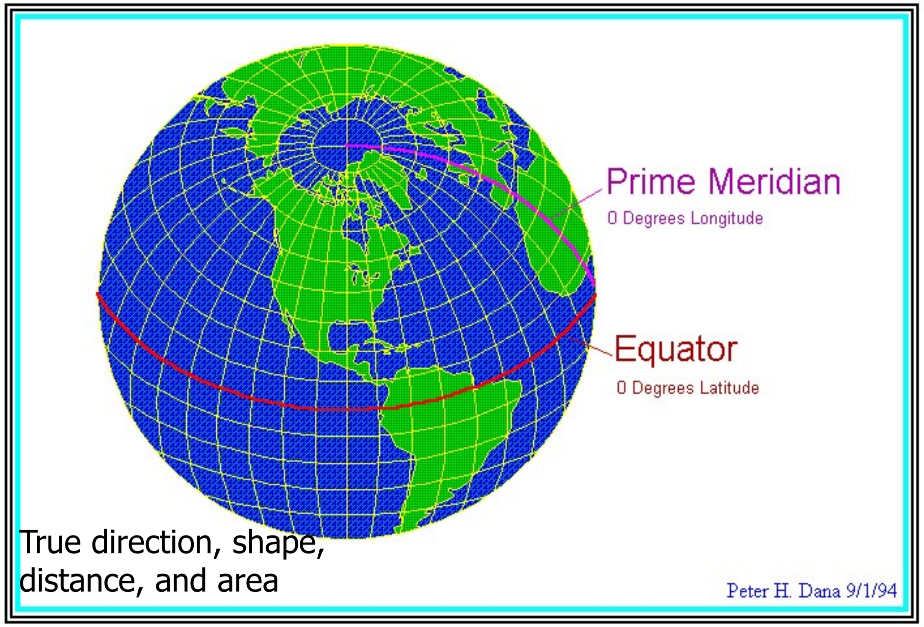
Globe

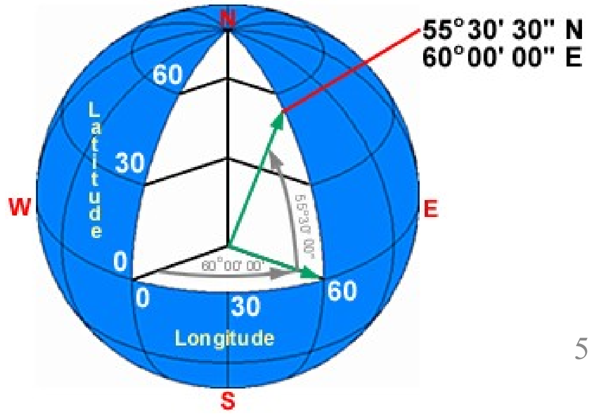
- Spherical Earth's surface

  -radius 6371 km
- Meridians (lines of longitude)

  - passing through Greenwich, England as prime meridian or 0º longitude.
- Parallels (lines of latitude)

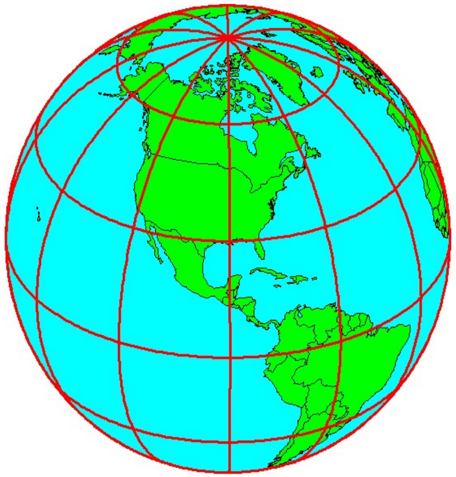
  - using equator as 0º latitude.
- degrees-minutes-seconds (DMS),
- decimal degrees (DD)



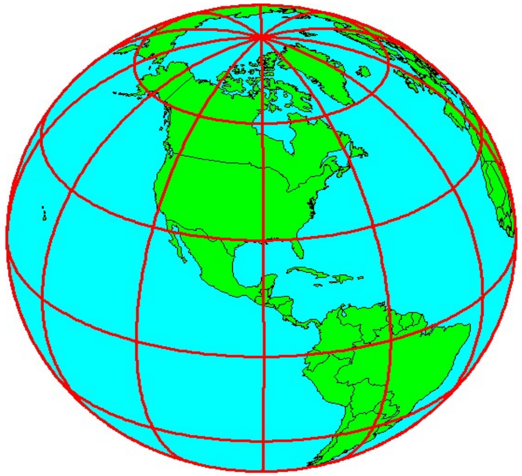


Shape of the Earth

We think of the earth as a sphere



It is actually a spheroid, slightly larger in radius at the equator than at the poles



Earth Shape Models

- **Flat earth models** are still used for plane surveying, over distances short enough so that earth curvature is insignificant (less than 10 km).

- **Spherical earth models** (Earth centered model) represent the shape of the earth with a sphere of a specified radius. Spherical earth models are often used for short range navigation and for global distance approximations. Spherical models fail to model the actual shape of the earth.

- **Ellipsoidal earth models** are required for accurate range and bearing calculations over long distances. Ellipsoidal models define an ellipsoid with an equatorial radius and a polar radius. The best of these models can represent the shape of the earth over the smoothed, averaged sea-surface to within about one-hundred meters.

- Although the earth is an ellipsoid, its major and minor axes do not vary greatly. The shape is so close to a sphere that it is often called a spheroid rather than an ellipsoid.

Shape of the Earth?

The Earth is "almost" a sphere with a circumference ~**40000 km**

Better approximate by an oblate ellipsoid

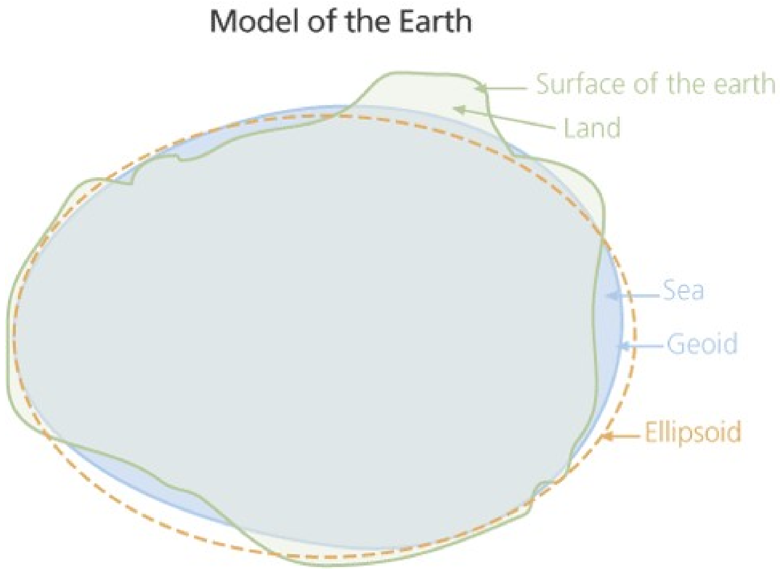
Today accepted Value	
Equatorial Radius	6378 km
Polar Radius	6357 km
Sphere of Equal Volume Radius	**6371 km**
Flattening	1/300 appx

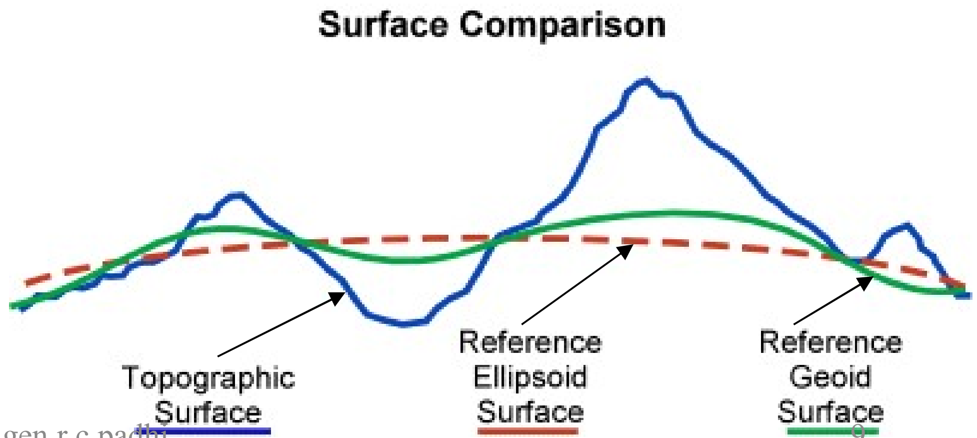
The "real" shape of the planet is approximated by the Geoid:

THE EQUIPOTENTIAL SURFACE AT THE MEAN SEA LEVELS

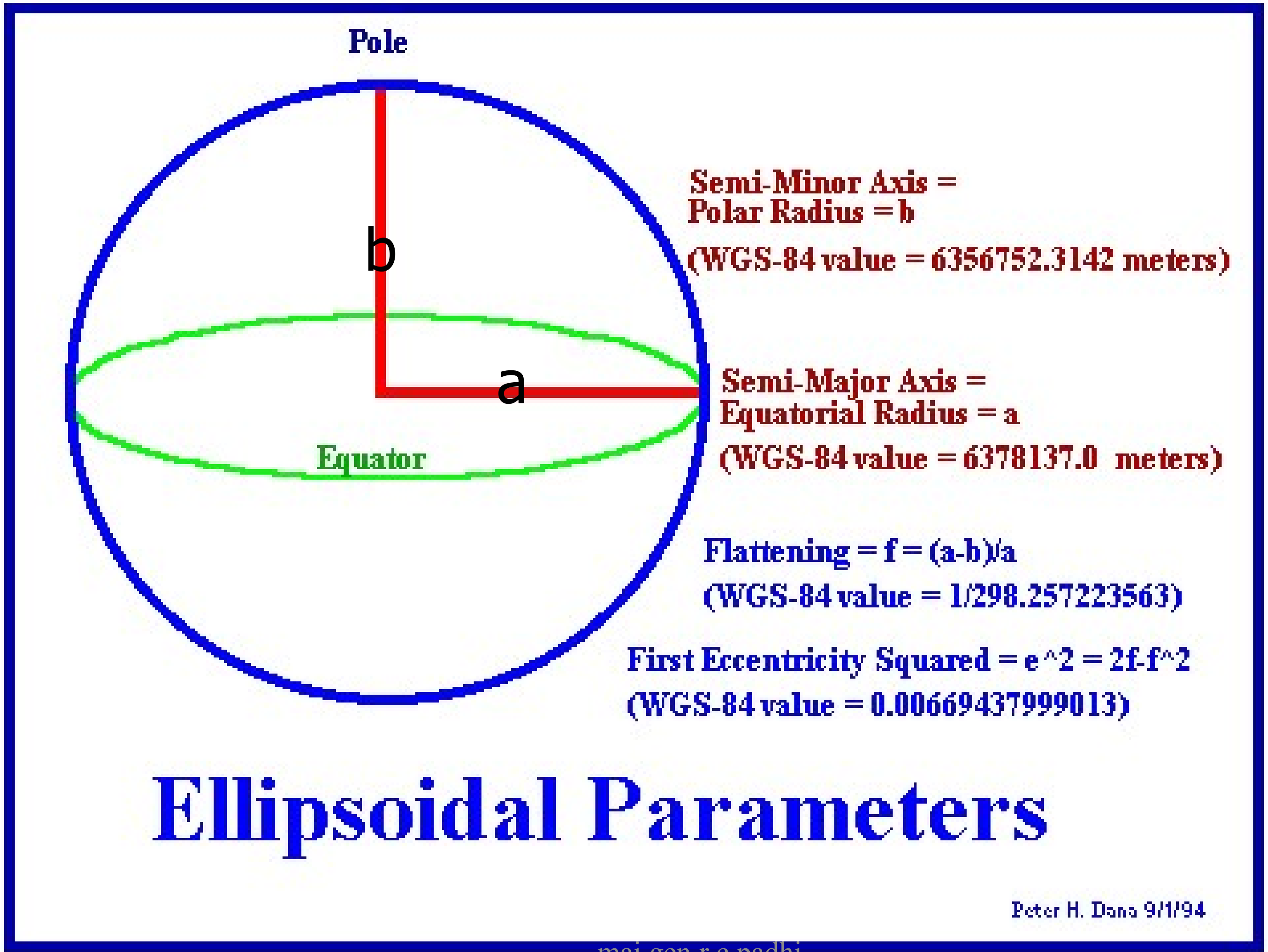
Earth Surface: Ellipsoid, Geoid, Topo

- The ***reference ellipsoid*** surface.**Everest,WGS 84**
- The ***reference geoid*** surface (mean sea level-MSL surface).
- The **real surface** of the the Earth (the ground) called the topographic surface.

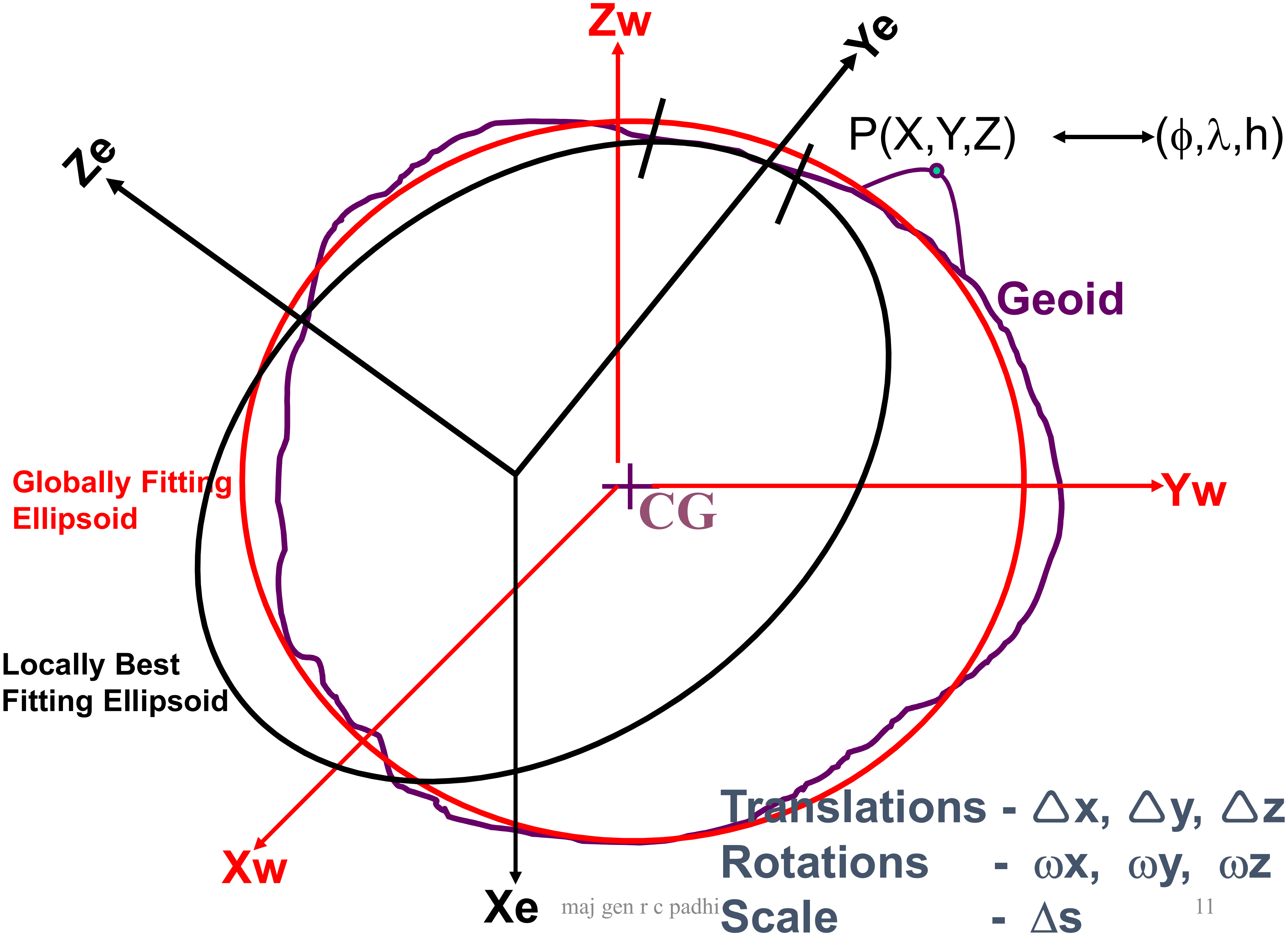




Ellipsoidal Parameters

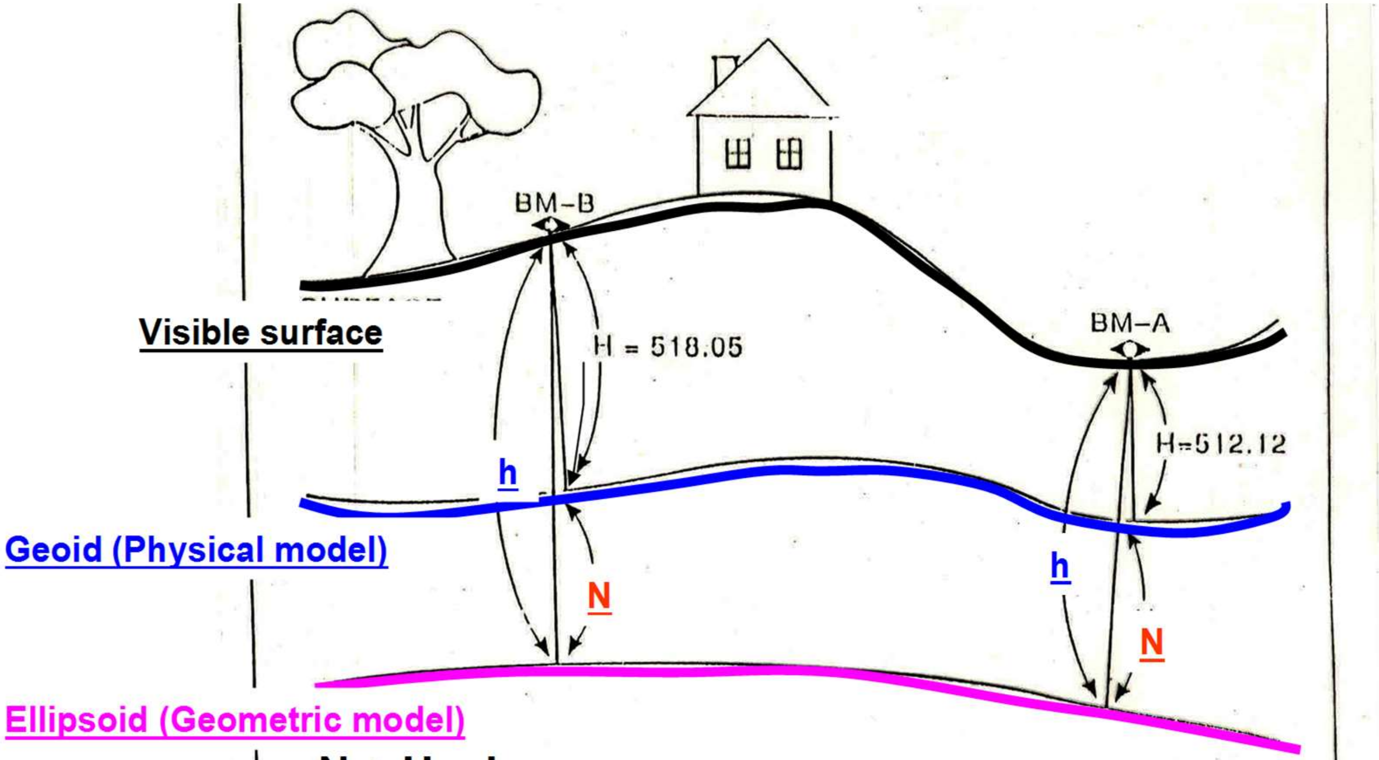


Geocentric & Locally Best Fitting Ellipsoids



Translations - △x, △y, △z
Rotations - ωx, ωy, ωz
Scale - Δs

Physical surface, geoid, ellipsoid



N + H = h

N – Geoidal separation which varies

H – Elevation / MSL Height (Orthometric)

h – Ellipsoidal or geodetic height

Accuracy of elevation of a point on earth's surface depends on accuracy of geoidal separation N.

GEODETIC SYSTEM

- **Geodetic system serves as the framework to determine co-ordinates of points with respect to the earth.**
- **Global Geodetic System such as WGS-84 is established using techniques of satellite geodesy.**
- **A geodetic (horizontal) datum has an earth-fixed reference ellipsoid that may have been fit, in some manner, to the surface of the earth in the area of interest. Geodetic datum's have parameters that define the size and shape of the ellipsoid.**
- **A vertical datum is a surface of zero elevation. Elevations are measured (Positive upward) from the vertical datum.**
- **Ideally, a vertical datum would closely approximate the geoid. It is impossible to access the geoid surface directly . Tide gauge measurements averaged over many years used to establish the local MSL.**

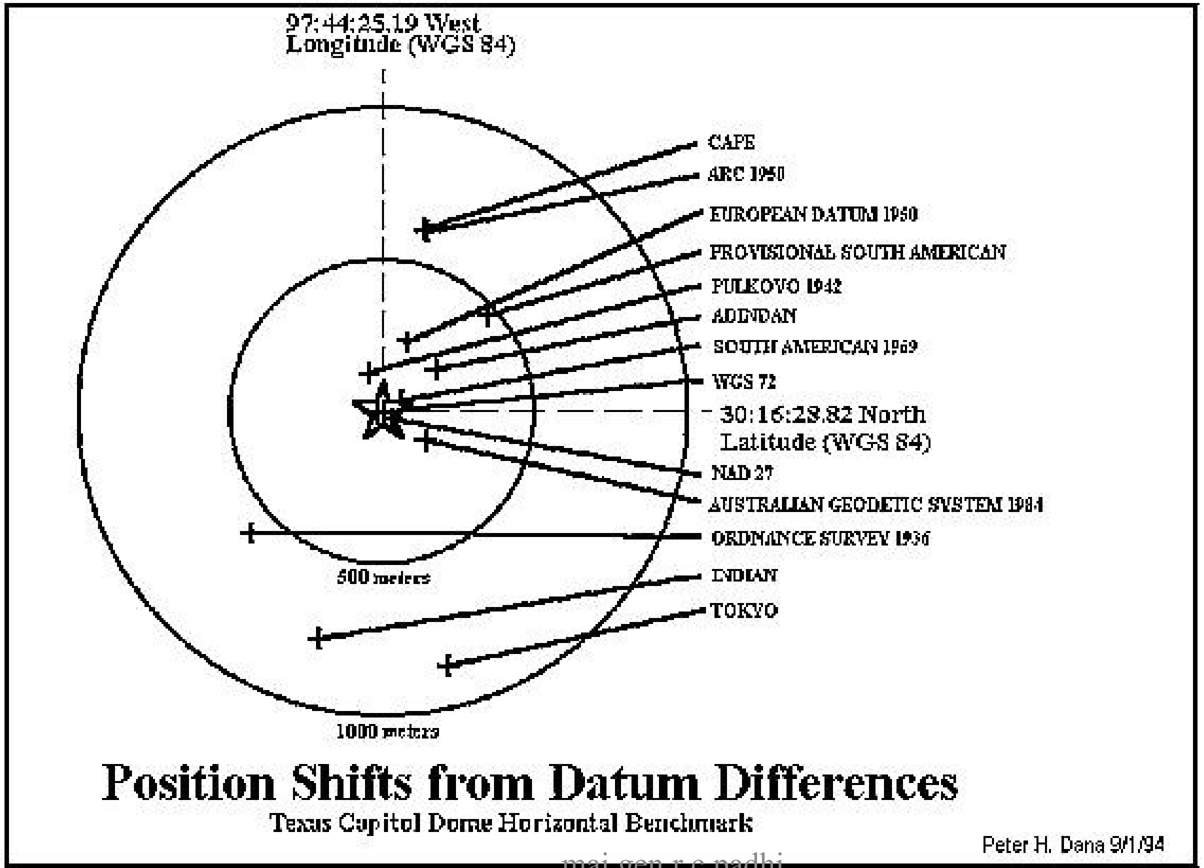
Horizontal Earth Datum

- An earth datum is defined by an ellipse and an axis of rotation
- NAD27 (North American Datum of 1927) uses the Clarke (1866) ellipsoid on a non geocentric axis of rotation
- NAD83 (NAD,1983) uses the GRS80 ellipsoid on a geocentric axis of rotation
- WGS84 (World Geodetic System of 1984) uses GRS80, almost the same as NAD83

GLOBAL DATUMS

- **The system which is truly universal is satellite based such as Global Positioning System (GPS).**

- **The ref system is a Geo-centric Ellipsoid.**



Selected Ellipsoids and Datums

Selected Reference Ellipsoids

Ellipse	Semi-Major Axis (meters)	1/Flattening
Airy 1830	6377563.396	299.3249646
Bessel 1841	6377397.155	299.1528128
Clarke 1866	6378206.4	294.9786982
Clarke 1880	6378249.145	293.465
Everest 1830	6377276.345	300.8017
Fischer 1960 (Mercury)	6378166.0	298.3
Fischer 1968	6378150.0	298.3
G R S 1967	6378160.0	298.247167427
G R S 1975	6378140.0	298.257
G R S 1980	6378137.0	298.257222101
Hough 1956	6378270.0	297.0
International	6378388.0	297.0
Krassovsky 1940	6378245.0	298.3
South American 1969	6378160.0	298.25
WGS 60	6378165.0	298.3
WGS 66	6378145.0	298.25
WGS 72	6378135.0	298.26
WGS 84	6378137.0	298.257223563

Peter H. Dana 9/1/94

Table 1: Datums and their principle areas of use

Datum	Area	Origin	Ellipsoid
WGS 1984	Global	Earth center of mass	WGS 84
NAD 1983	North America, Caribbean	Earth center of mass	GRS 80
NAD 1927	North America	Meades Ranch	Clarke 1866
European 1950	Europe, Middle East, North Africa	Potsdam	International

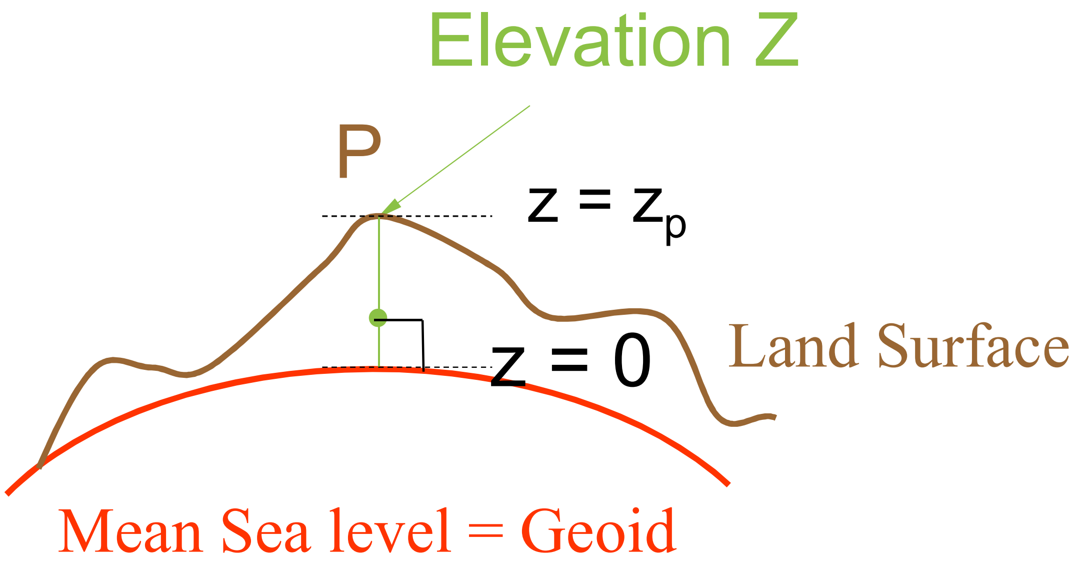
Source: http://maic.jmu.edu/sic/standards/datum.htm

Vertical Earth Datum

- A vertical datum defines elevation, z
- EVEREST (Indian MSL based on average tidal observation of Indian coast lines).
- NGVD29 (National Geodetic Vertical Datum of 1929)
- NAVD88 (North American Vertical Datum of 1988)

takes into account a map of gravity anomalies
between the ellipsoid and the geoid

Definition of Elevation



Elevation is measured from the Geoid

TRADITIONAL SURVEY

Horizontal positioning

TRIANGULATION

TRILATERATION

TRAVERSING

ASTRONOMICAL POSITIONING

Vertical positioning

GEODETIC LEVELING

TRIGONOMETRIC HEIGHTING

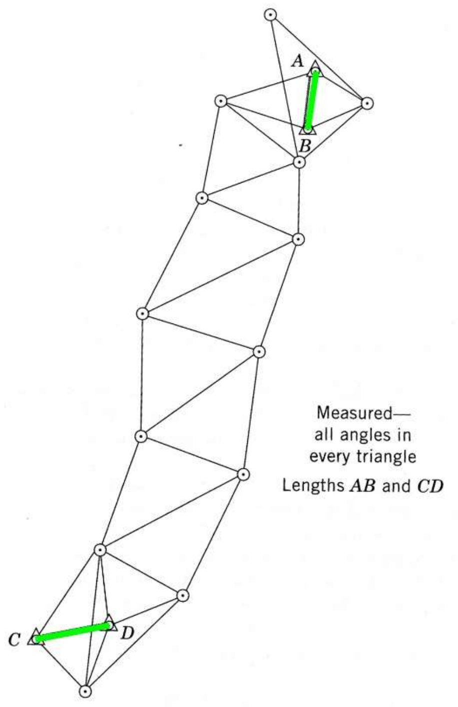
BAROMETRIC LEVELING

GPS /DGPS, ETS ,LiDAR for both horizontal & vertical

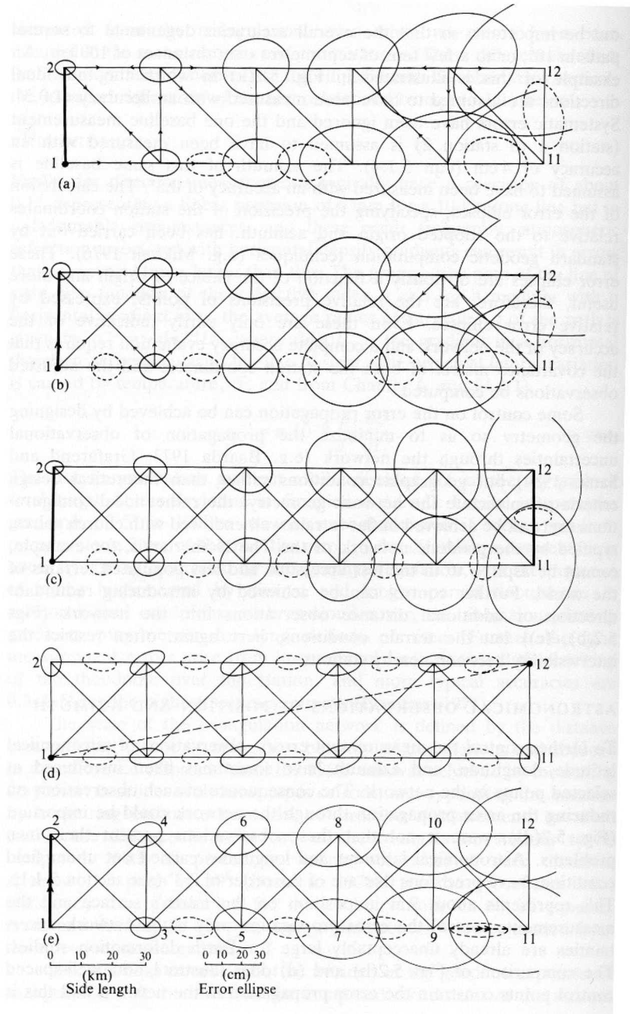
TRADITIONAL SURVEY

Horizontal positioning

TRIANGULATION



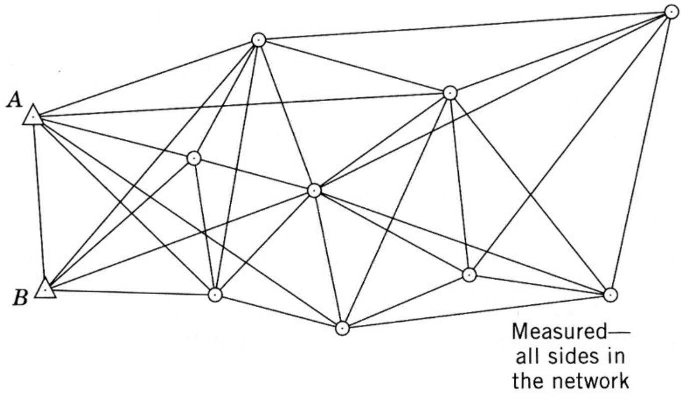
25. Triangulation Network



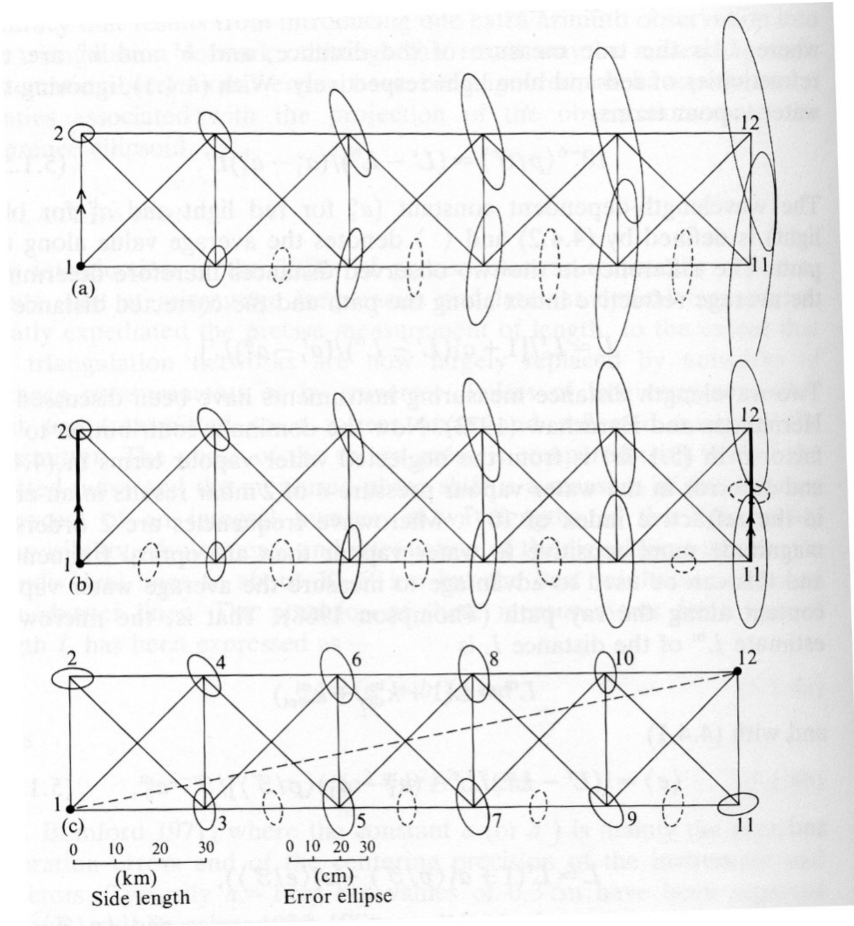
TRADITIONAL SURVEY

Horizontal positioning

TRILATERATION



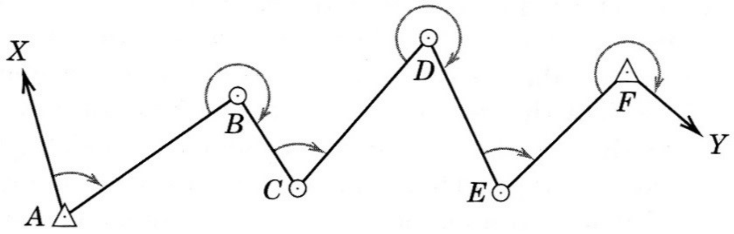
27. A Trilateration Network



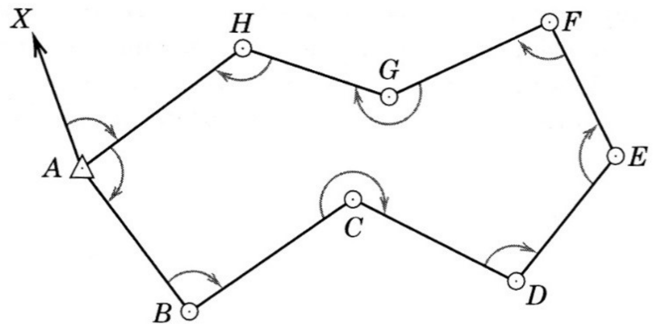
TRADITIONAL SURVEY

Horizontal positioning

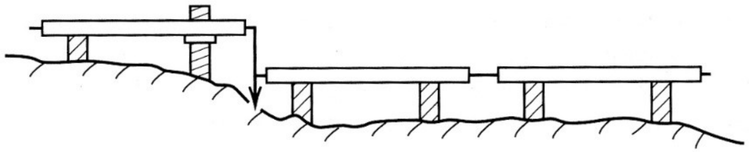
TRAVERSE



Closed Traverse



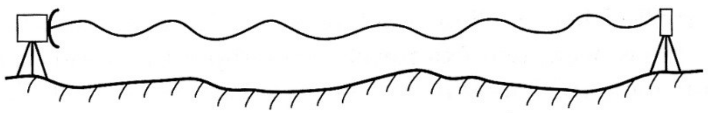
28. Traverses



(a) 18th and 19th Century — by wooden bars

(b) First Half 20th Century — by invar wire in catenary

EDM



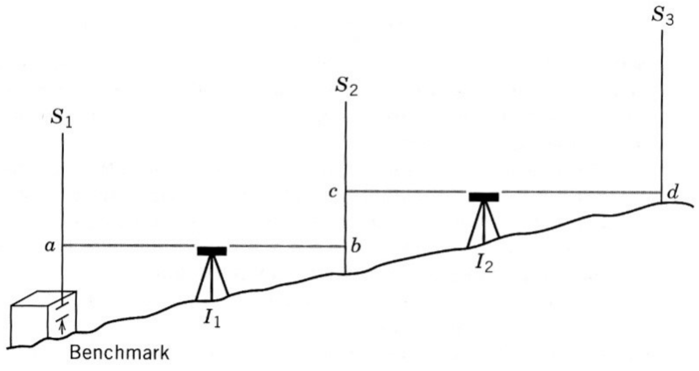
(c) Today — by electromagnetic distance measuring

26. Equipment for Distance Measurement

TRADITIONAL SURVEY

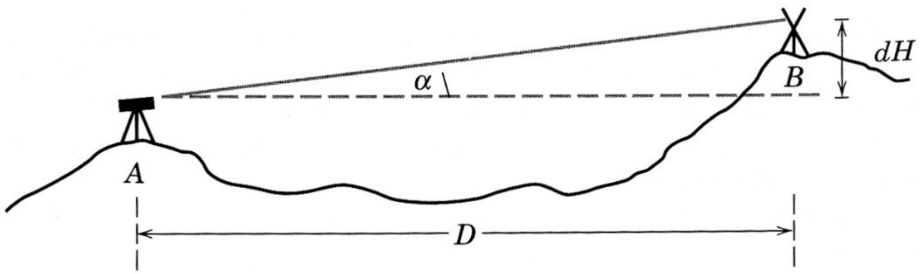
Vertical positioning

GEODETIC LEVELING



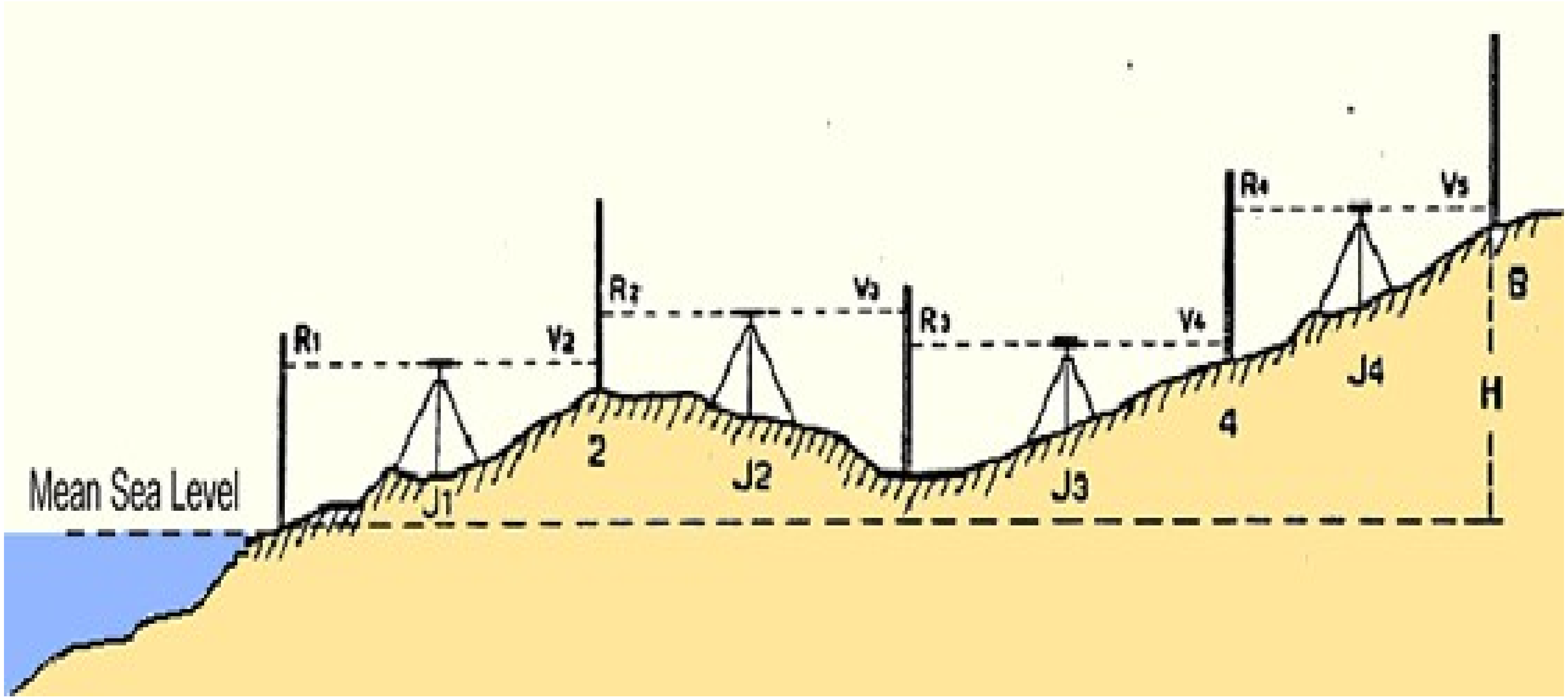
(a) Spirit Leveling

TRIGONOMETRIC HEIGHTING



(b) Trigonometric Leveling (Simplified)

Levelling

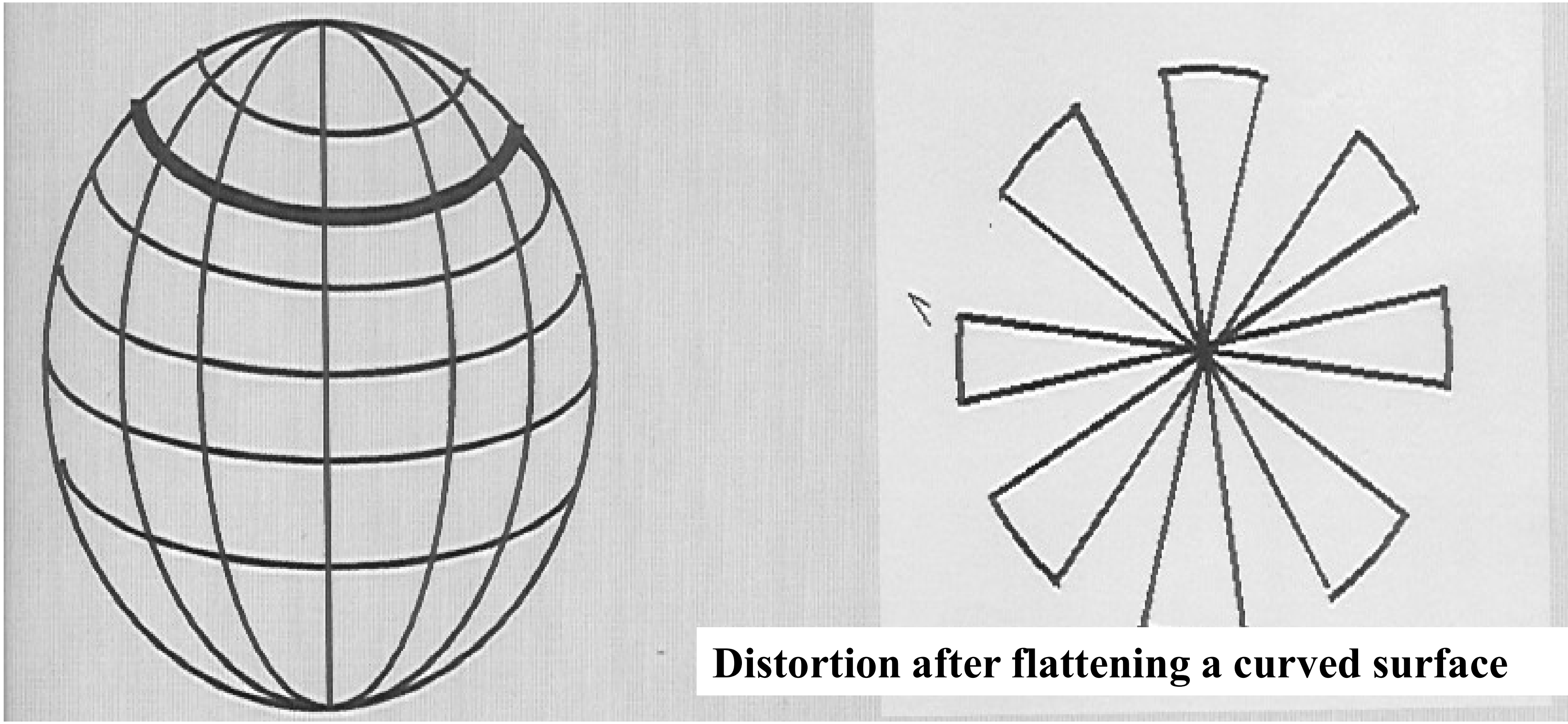


Differential leveling for height measurements

(Mean Sea Level is the starting point for the height measurements)

DEFORMATION

- The transformation from the curved surface of the Earth to the flat plane of the map is never completed successfully.



Distortion after flattening a curved surface

Map projection can be:- Conformal (Shape & angle)

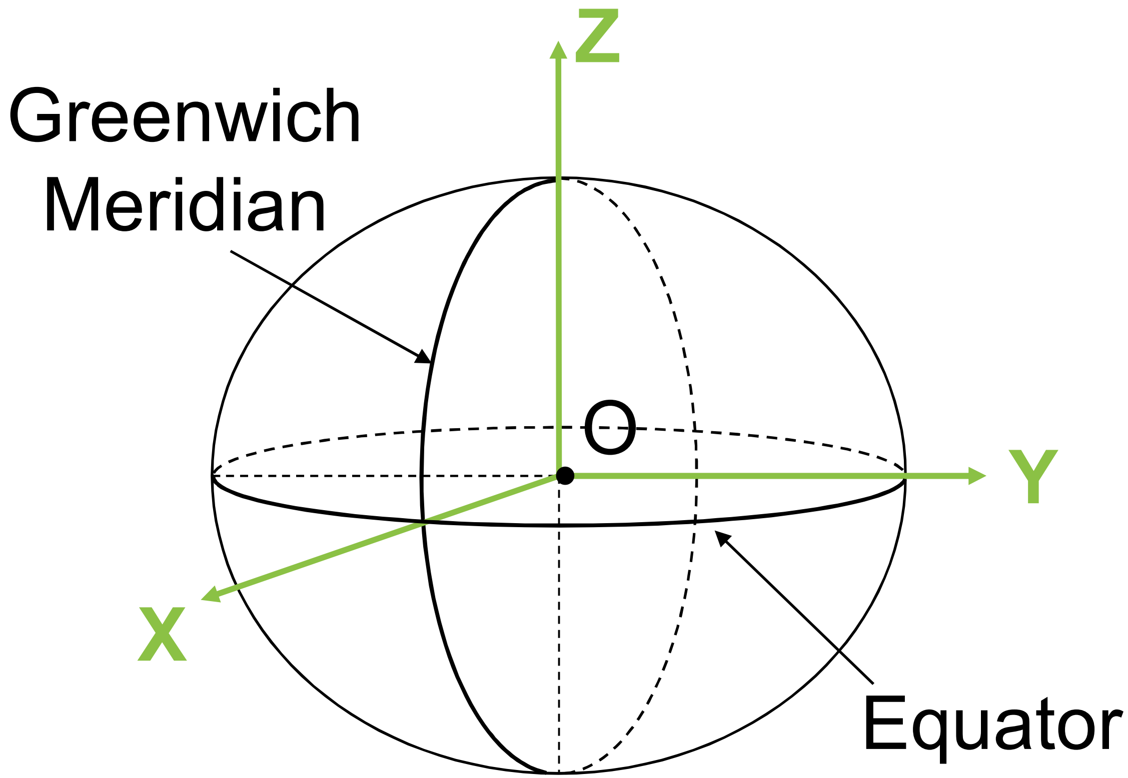
Equal Area

Equidistance

Coordinate System types

- Global Cartesian coordinates (x,y,z) for the whole earth
- Geographic coordinates (ϕ, λ, z)
- Projected coordinates (x, y, z) on a local area of the earth's surface
- The z-coordinate in global and projected is defined geometrically; in geographic the z-coordinate is defined gravitationally.

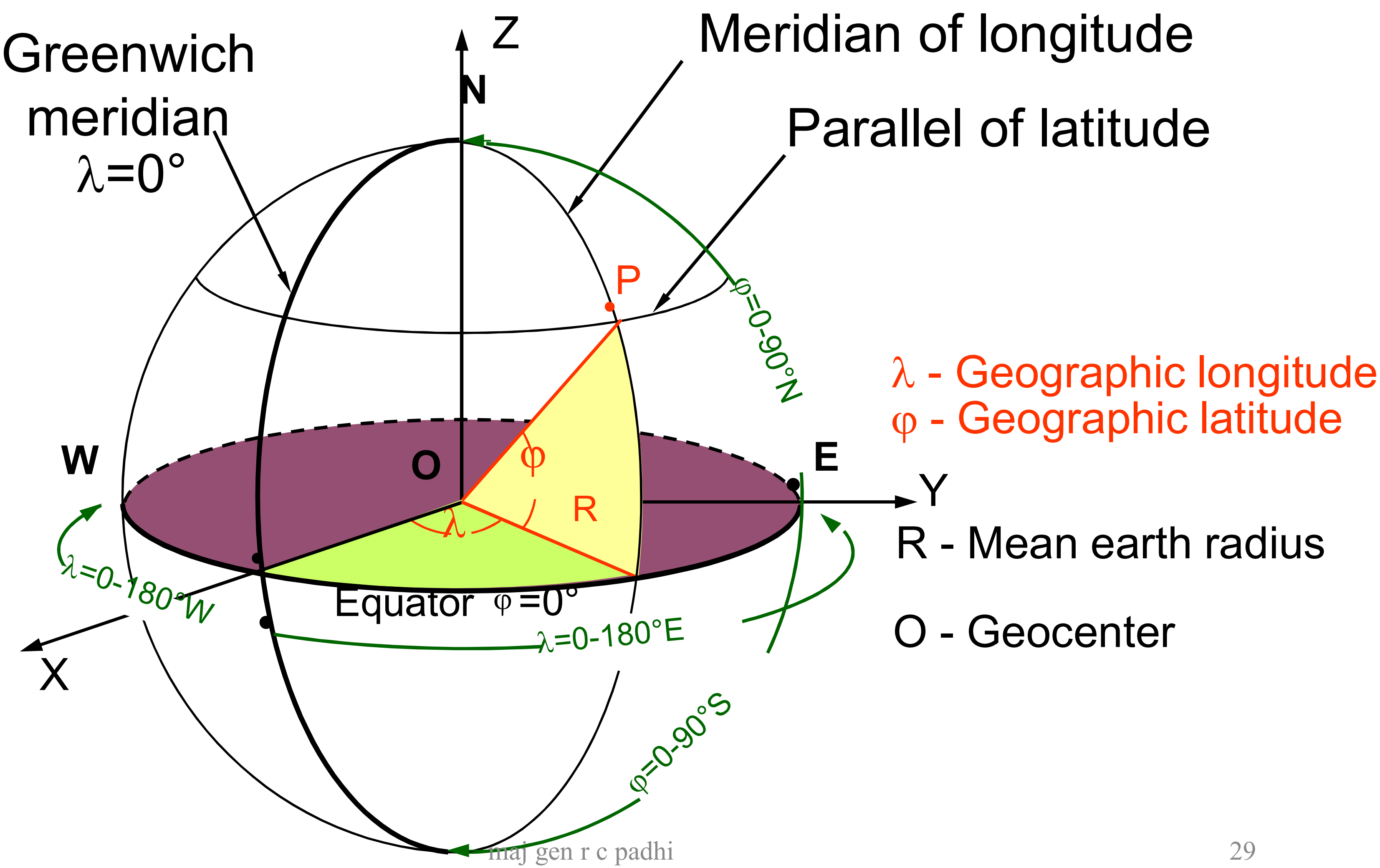
Global Cartesian Coordinates (x,y,z)



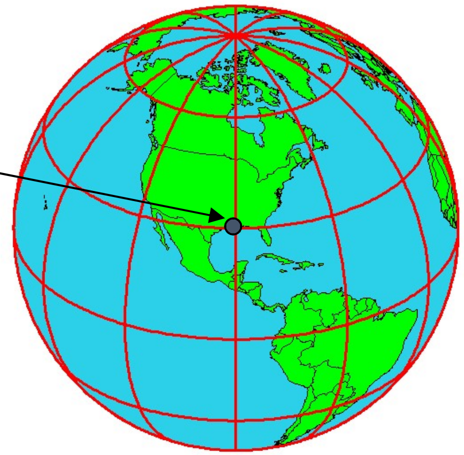
Geographic Coordinates (ϕ, λ, z)

- Latitude (ϕ) and Longitude (λ) defined using an ellipsoid, an ellipse rotated about an axis
- Elevation (z) defined using geoid, a surface of constant gravitational potential
- Earth datum define standard values of the ellipsoid and geoid

Latitude and Longitude on a Sphere



Example: What is the length of a 1º increment on a meridian and on a parallel at 30N, 90W? Radius of the earth = 6370 km.



Solution:

- A 1º angle has first to be converted to radians

π radians = 180 º, so 1º = $\pi/180$ = 3.1416/180 = 0.0175 radians

- For the meridian, $\Delta L = R_e \, \Delta\phi = 6370 * 0.0175 =$ <u>111 km</u>

- For the parallel, $\Delta L = R_e \, \Delta\lambda \, \text{Cos}\ \phi$

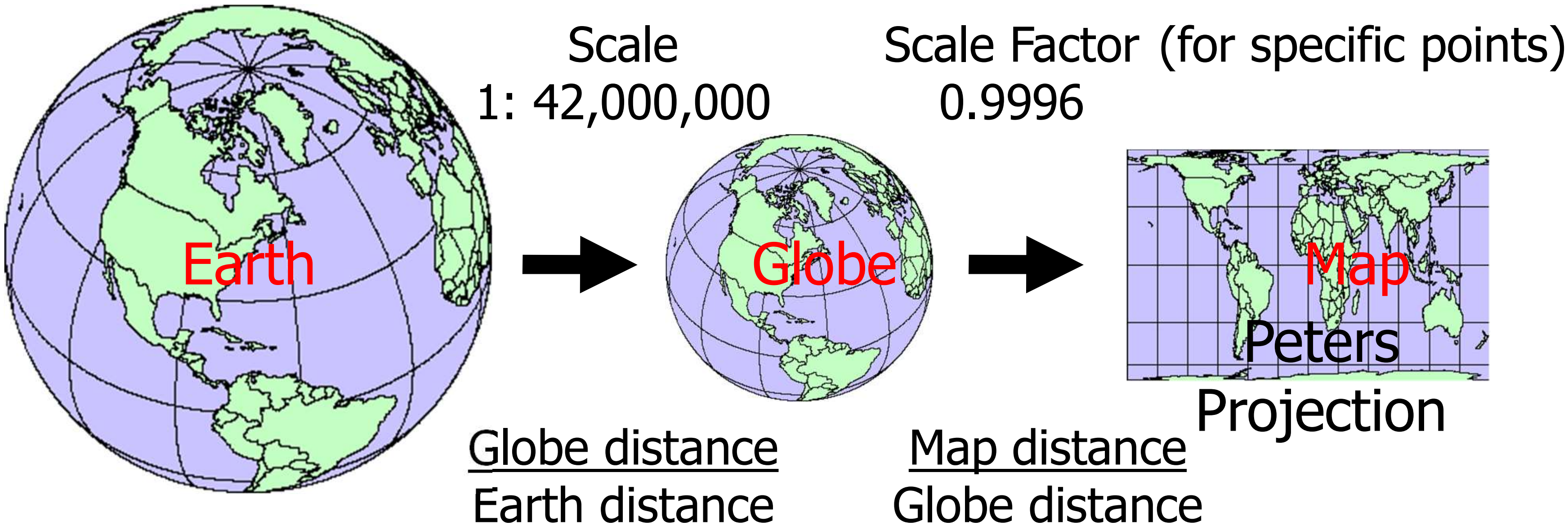
$= 6370 * 0.0175 * \text{Cos}\ 30$

$=$ <u>96.5 km</u>

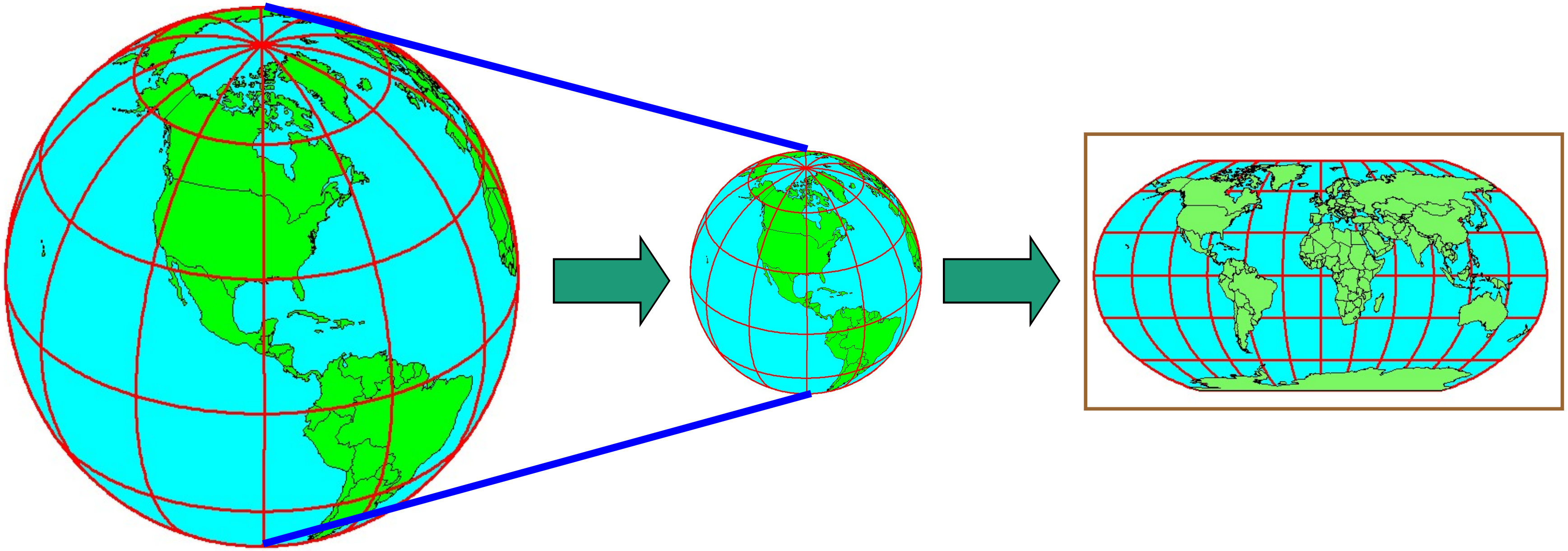
- Parallels converge as poles are approached

Projecting the earth flat

- Projections – transformation of curved earth to a flat map; systematic rendering of the latitude & longitude graticules to rectangular coordinate system.



Earth to Globe to Map



Map Scale:

Representative Fraction

$$= \frac{\text{Globe distance}}{\text{Earth distance}}$$

(e.g. 1:24,000)

Map Projection:

Scale Factor

$$= \frac{\text{Map distance}}{\text{Globe distance}}$$

(e.g. 0.9996)

Map Projections: Basics

- A map projection is a mathematical model for conversion of locations from a three-dimensional earth surface to a two-dimensional map representation.
- This conversion distorts some aspect of the earth's surface, area, shape, distance, or direction.
- Every projection has its advantages and disadvantages. There is no "best" projection. Some distortions of conformity (shape), scale, distance, direction, and area .
- Some projections minimize distortions in some of these properties at the expense of maximizing errors in others. Some projection are attempts to moderately distort all properties
- We select the one best suited to the needs, reducing distortion of the most important features.

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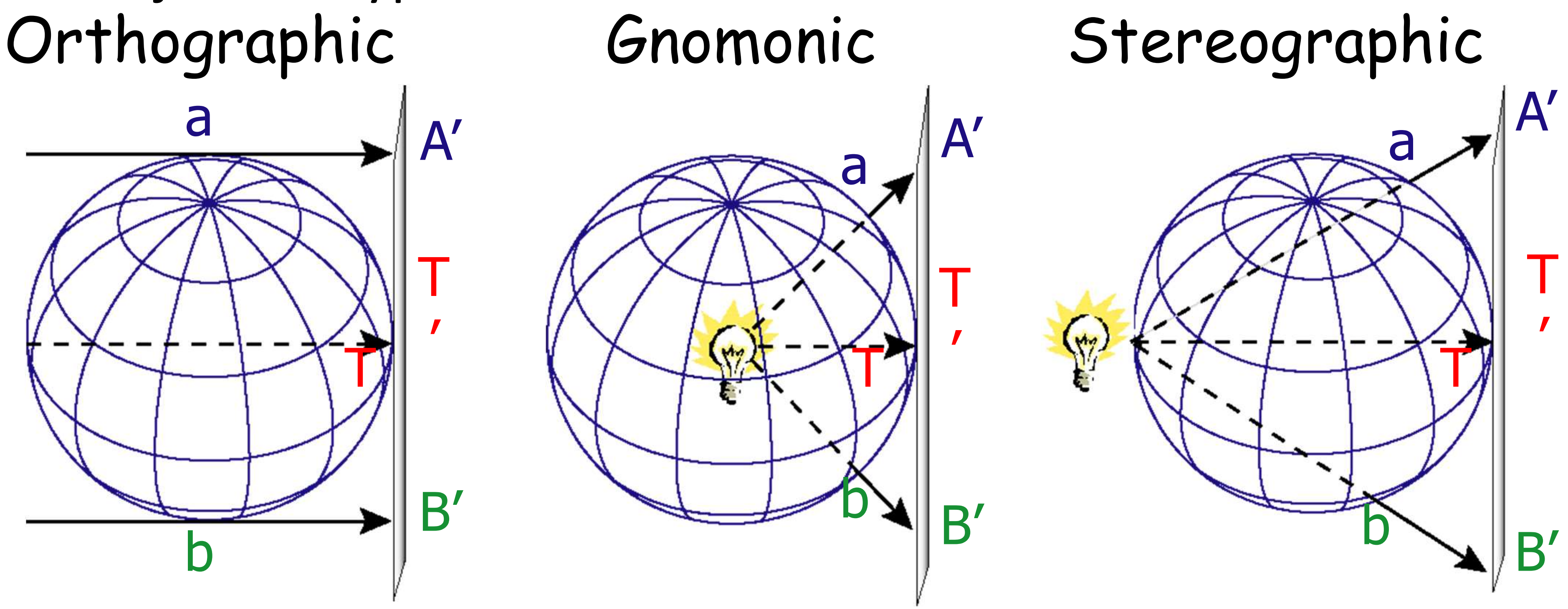
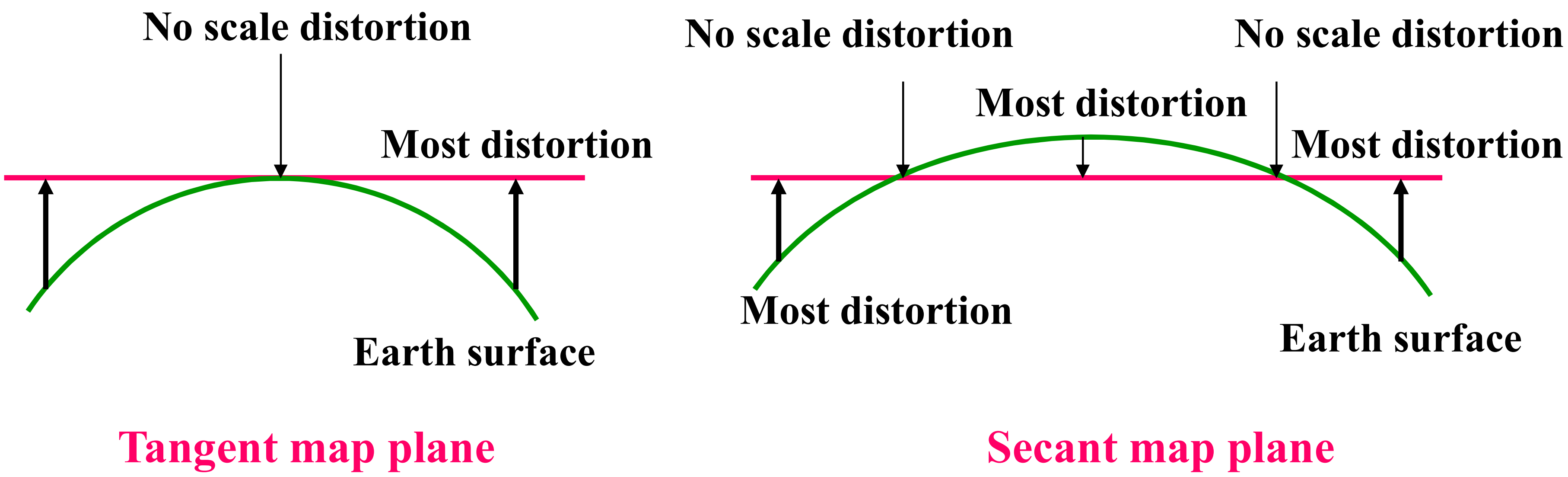
Projecting the earth flat

Projection types:



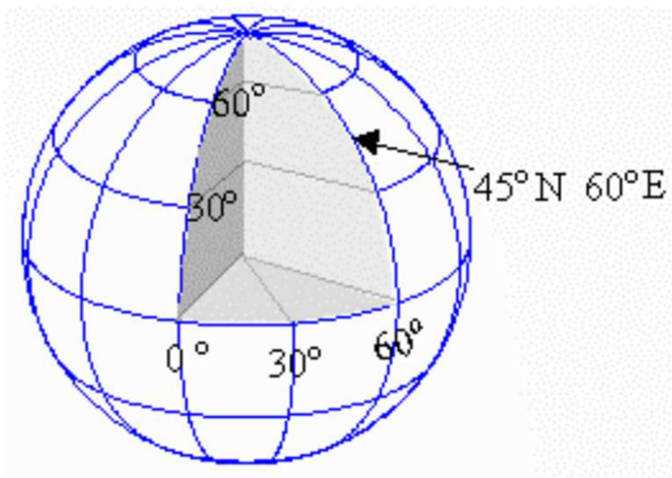
SCALE DISTORTION

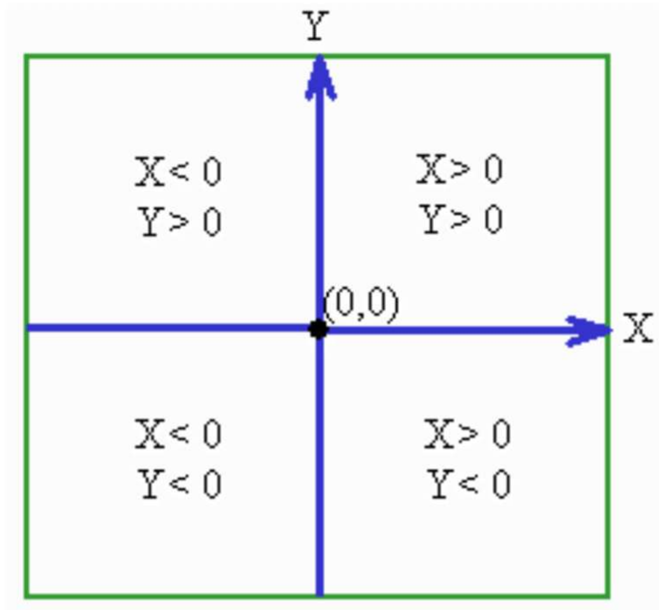
- It is impossible to project the Earth on a piece of paper without location distortions.



- It is important to know the extent to which the scale varies on map.

Geographic and Projected Coordinates





(ϕ, λ) ⟷ (x, y)

Map Projection

Types of Projections

- Conic (Albers Equal Area, Lambert Conformal Conic) - good for East-West land areas.
- Cylindrical (Transverse Mercator) - good for North-South land areas.
- Azimuthal (Lambert Azimuthal Equal Area) - good for global views

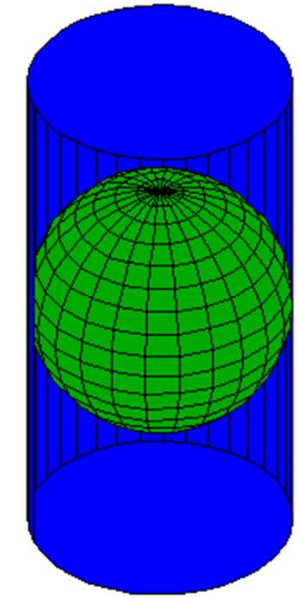
Classes of Map projections

Physical models:

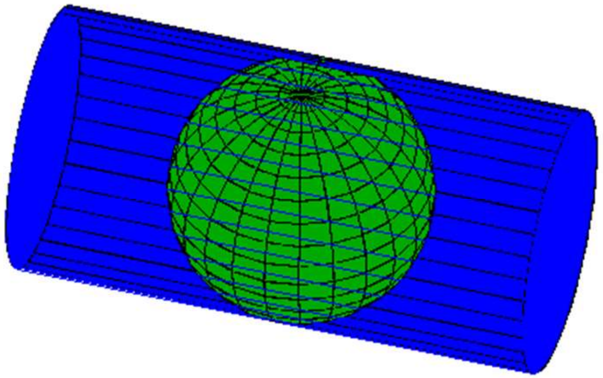
- Cylindrical projections (cylinder)
  - Tangent case
  - Secant case
- Conic Projections (cone)
  - Tangent case
  - Secant case
- Azimuthal or planar projections (plane)
  - Tangent case
  - Secant case

Distortion properties:

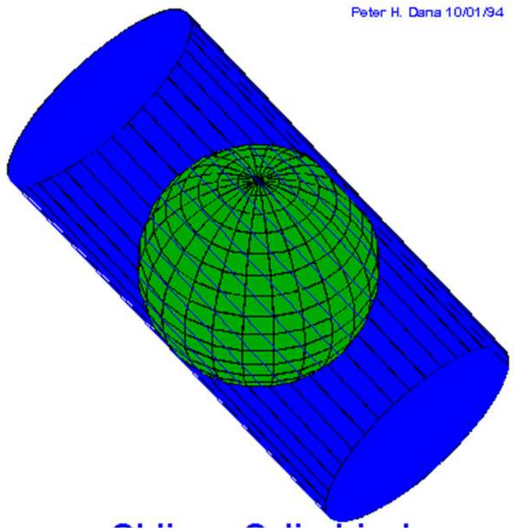
- Conformal (preserves local angles and shape)
- Equal area or equivalent (area)
- Equidistant (scale along a center line)
- Azimuthal (directions)



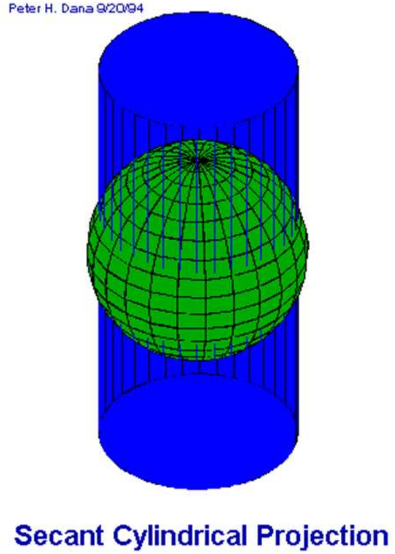
Cylindrical



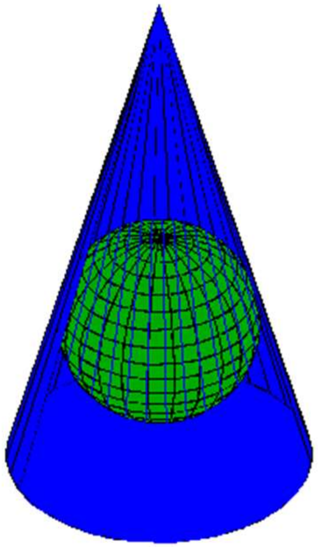
Transverse Cylindrical



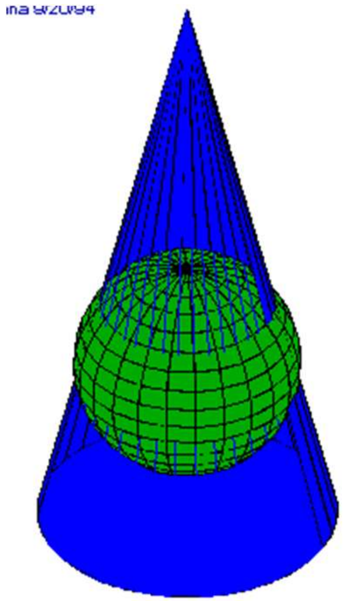
Oblique Cylindrical



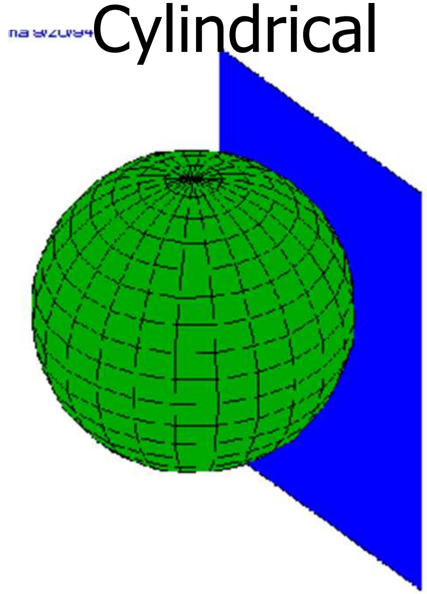
Secant Cylindrical



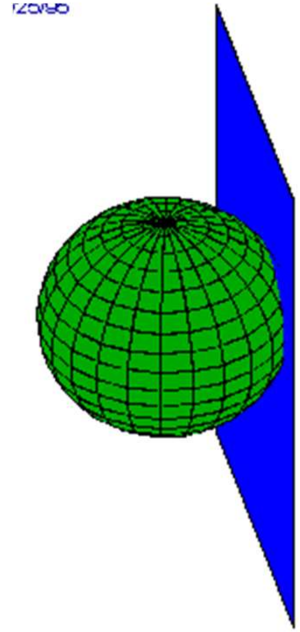
Conical



Secant Conical



Planar



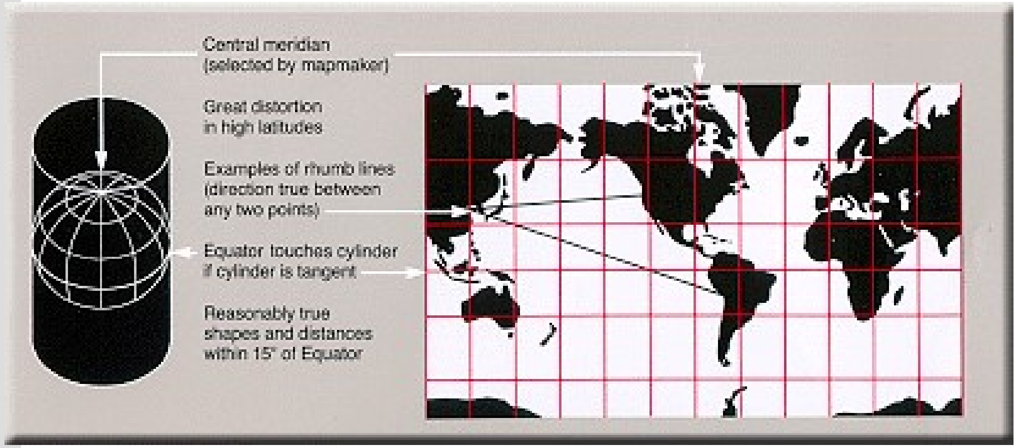
Secant Planar

Commonly Used Projections

- **Mercator Projection (1569)**

  - Directions are true along straight line of any two points, Distances are true only along equator, and reasonable correct within 15 of equator, in secant model, distance along two parallels are correct in scale instead of the Equator. Areas and shapes of large area are distorted. Distortion increases away from Equator and is extreme in polar regions. However, map is **conformal** in that angles and shapes within any small area is essentially true.

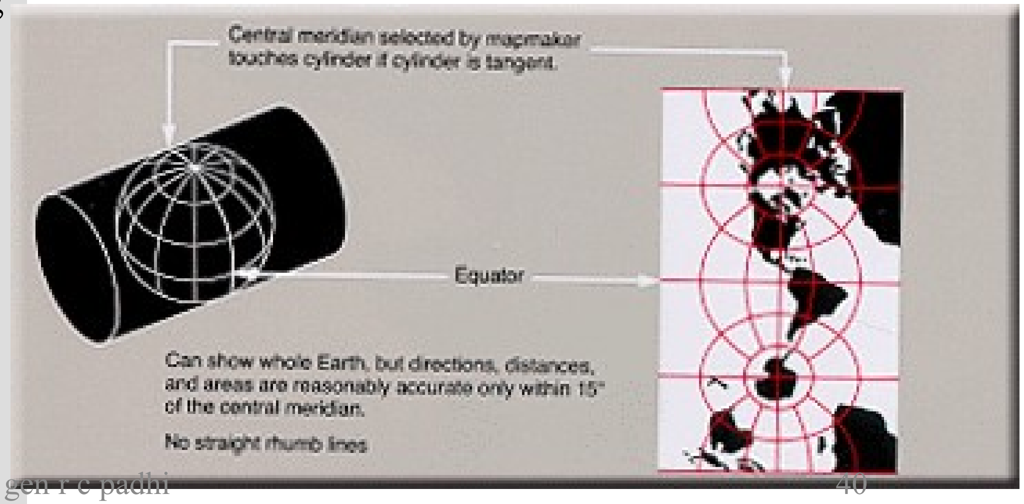
  - Used for navigation or maps of equatorial regions.



- **Transverse Mercator Projection (Lambert 1772)**

  - Distances are true *only* along the central meridian selected by the mapmaker or else along two lines parallel to it, but all distances, directions, shapes, and areas are reasonably accurate within 15° of the central meridian. Distortion of distances, directions, and size of areas increases rapidly outside the 15° band. Because the map is **conformal**, however, shapes and angles within any small area (such as that shown by a USGS topographic map) are essentially true.

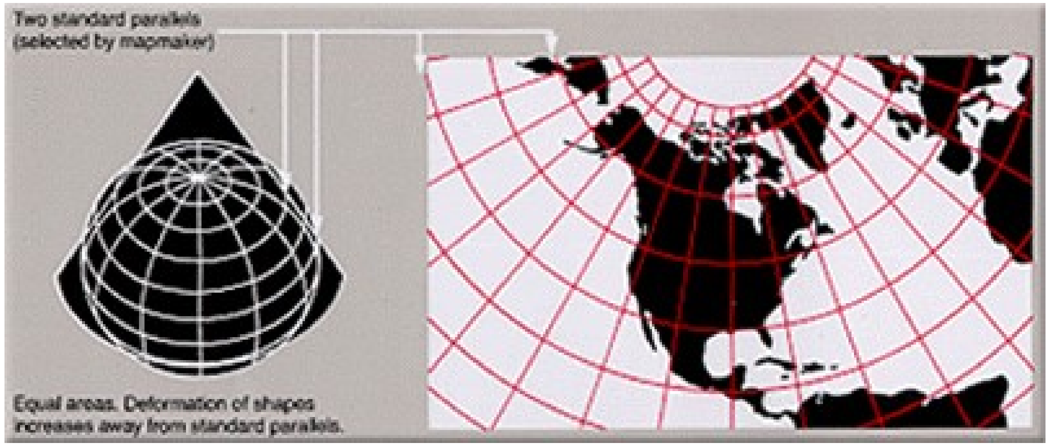
  - Used for many USGS 1:24,000 to 1:250,000 map



Commonly Used Projections

- **Albers Equal-Area Conic Projection**

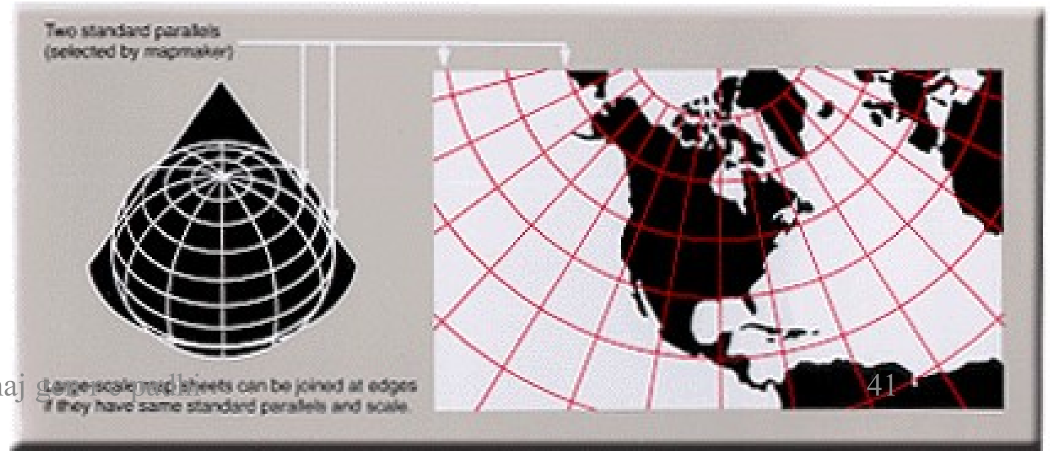
  - **All areas** on the map are proportional to the same areas on the Earth. **Directions** are reasonably accurate in limited regions. **Distances** are true on both standard parallels. Maximum scale error is 1 1/4% on map of conterminous States with standard parallels of 29 1/2°N and 45 1/2°N. **Scale** true only along standard parallels .



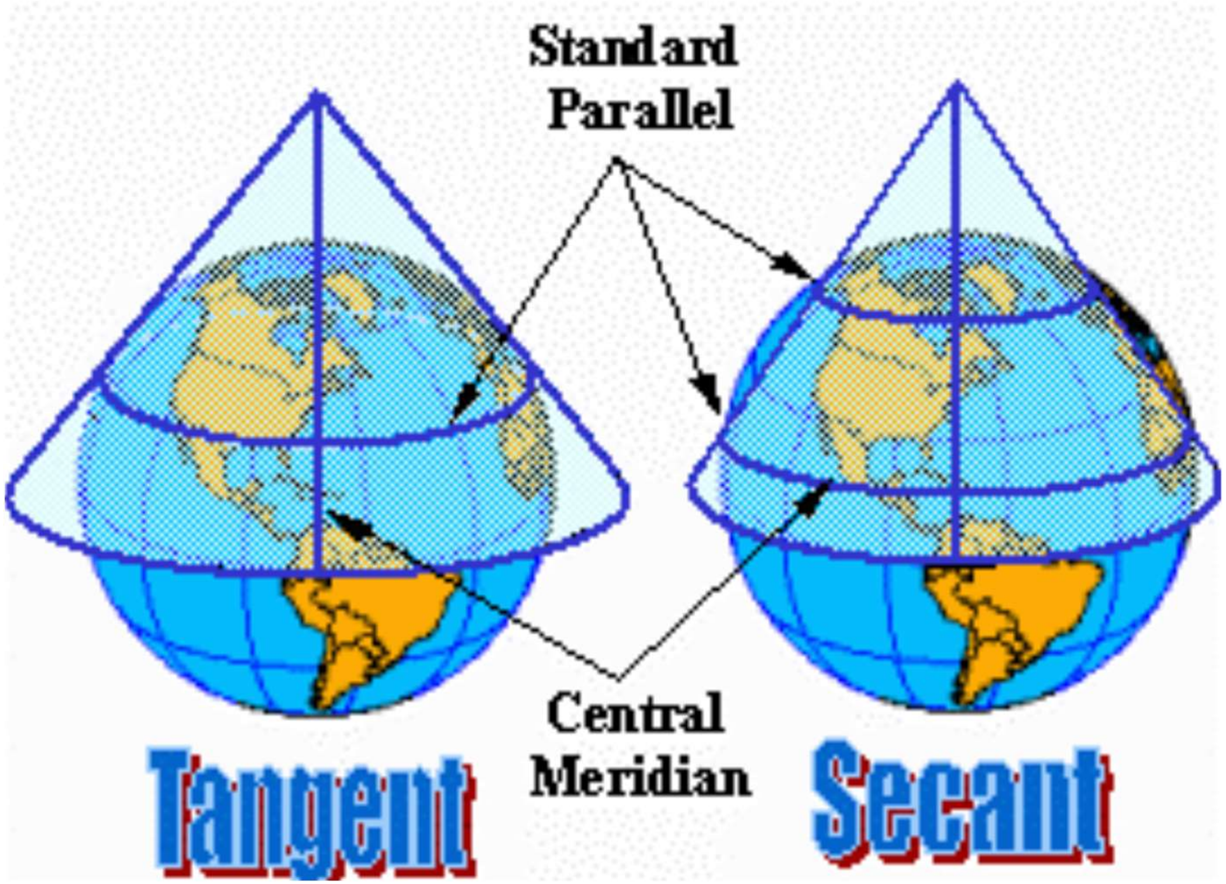
Lambert Conformal Conic/polyconic Projection.

- **Distances** true *only* along standard parallels; reasonably accurate elsewhere in limited regions. **Directions** reasonably accurate. **Distortion** of shapes and areas minimal at, but increases away from standard parallels. **Shapes** on large-scale maps of small areas essentially true Used for Indian TOPO maps, standard parallels vary zone wise.

- Used for topographic maps.

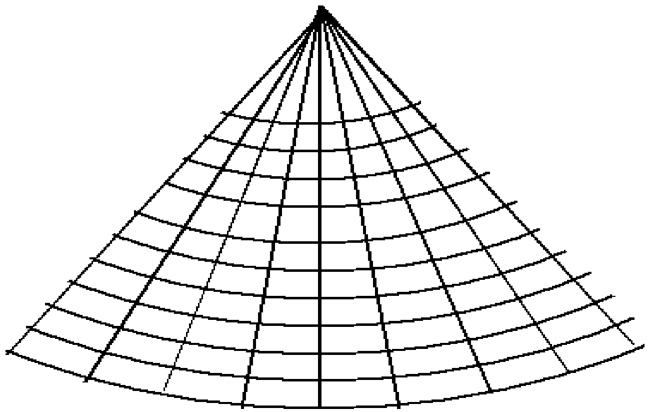


Conic Projections
(Albers, Lambert)

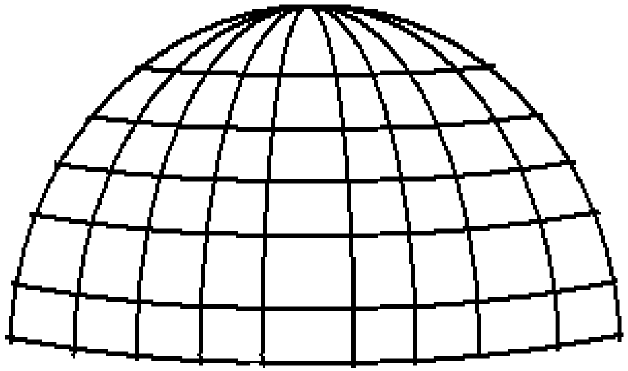


POLYCONIC PROJECTION

- **Class : Neither Conformal nor equal area.**
- **Projection principle : It consists of many (poly) cones involved. The cones are tangent to each parallel, so that meridians are curved & parallels are also curved.**



Conic Projection



Polyconic Projection

Uses :- 1. All topographic maps in India are in Polyconic Projection.
2. Used for large scale mapping of USA till 1950 only.
3. Not a suitable projection for digital environment due to problem of rolling fit .

POLYCONIC PROJECTION FOR INDIAN MAPS

- All topographical maps on scale 1:25,000 ; 1:50,000 and 1:250,000 are prepared on Polyconic Projection.
- Projection Parameters of Polyconic Projection are :
  - **Longitude of Origin ………………..**
  - **Latitude of Origin ………………..**
  - **False Easting ………………..**
  - **False Northing ………………..**
  - **Scale Reduction Factor ………………..**
- For each SOI map sheet, the center of map has been taken as origin. It means each sheet is being projected individually and is having identical coordinates of corner points.
- Hence these maps can not be joined seamlessly in digital environment.

Maps in Geographic Coordinates

- **Digital map data can also be stored in 'Geographic coordinates i.e. Latitude and Longitude.**

- **Geographic coordinate system is popular in digital environment where contiguous maps are displayed on screen and measurements made in GIS environ.**

- **1:250,000 (1° x 1°) or 1:50,000(15'x15') scale maps get displayed as perfect squares.**

- **Most popular GIS system has option for map viewing in geographic system (un projected) or projected.**

Limitations of Polyconic Projection

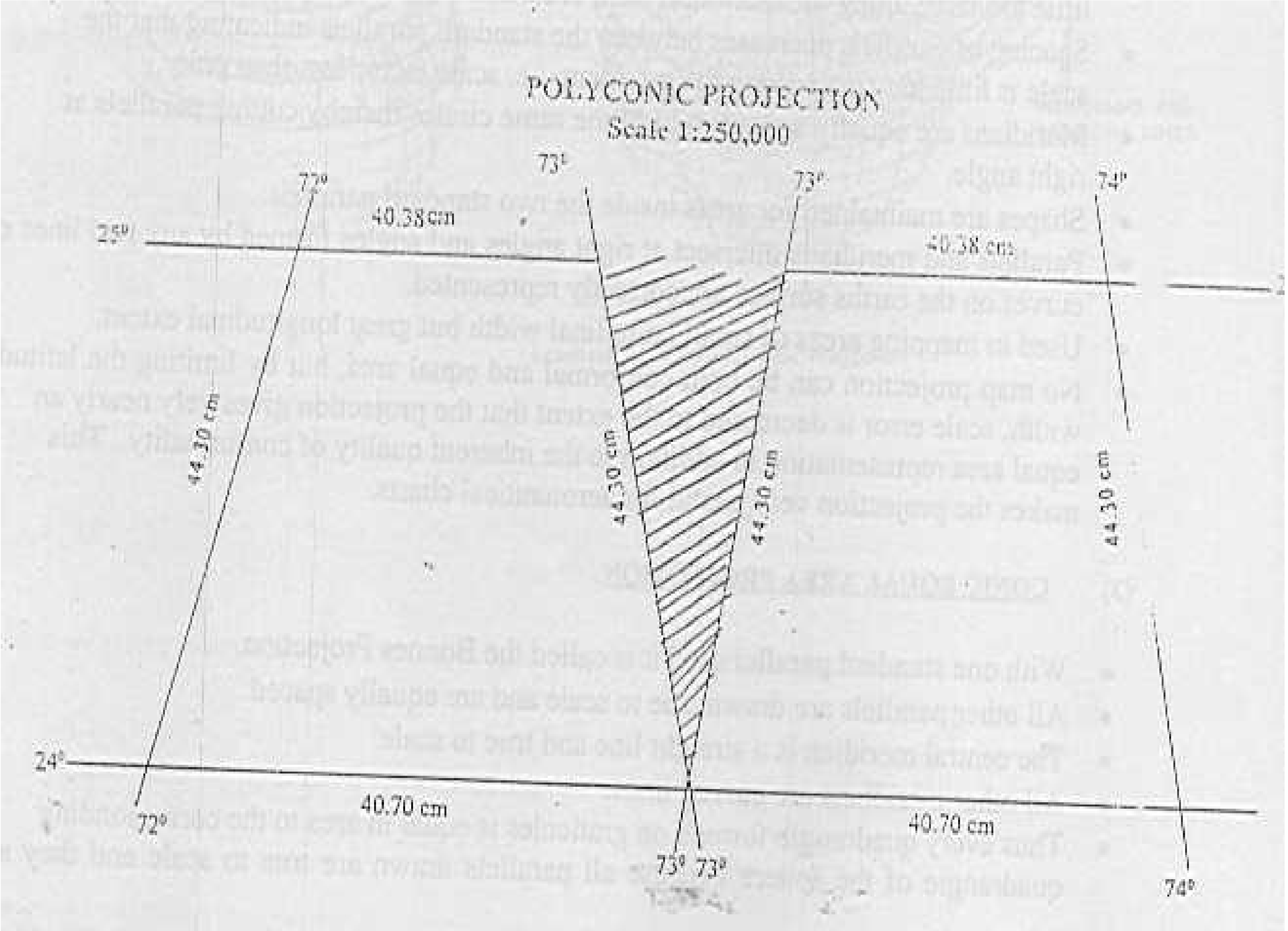
- **Since each topographical map sheet is projected independently with origin at the center of the map, two or more digital map sheets cannot be joined seamlessly, unless they are transformed to some common coordinate systems.**

- **Shapes of the maps is trapezoidal therefore even the raster maps cannot be joined in multiple directions.**

- **Distortions increases as the distance from central meridian increases.**

- **Not suitable for digital environment. USGS used this projection for their topo sheets till 1957 only.**

POLYCONIC PROJECTION

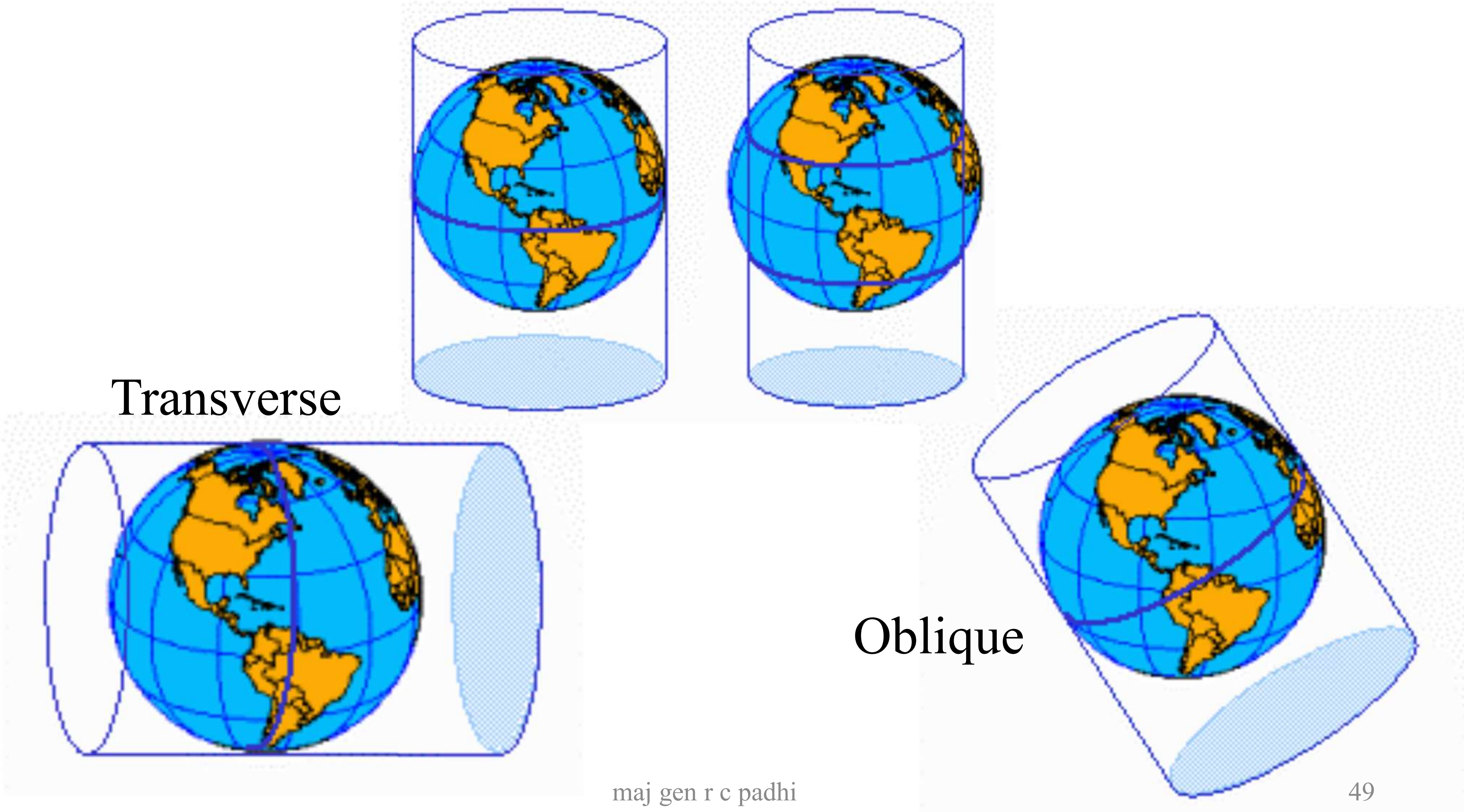


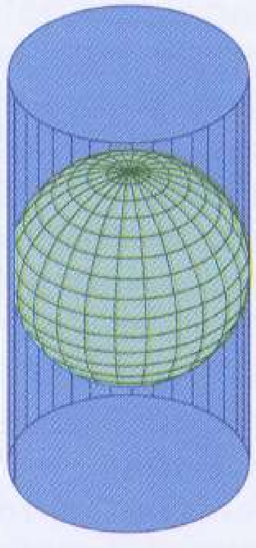
Limitations of Everest Ellipsoid

- **The center of the Everest ellipsoid does not coincide with the center of earth and is not the best fit for the globe.**
- It fits well with earth for small area around India
- **Even in some portions of India , it does not fit properly.**
- **WGS84 is the best fit system for the globe.**
- **GPS coordinate are in the WGS-84 system.**
- **At present, to use GPS coordinates with pre 2005 topographical maps, these need to be transformed to Everest ellipsoid from WGS-84.**
- **For such transformation of coordinates, 7 transformation parameters are needed. SOI has worked out these parameters.**

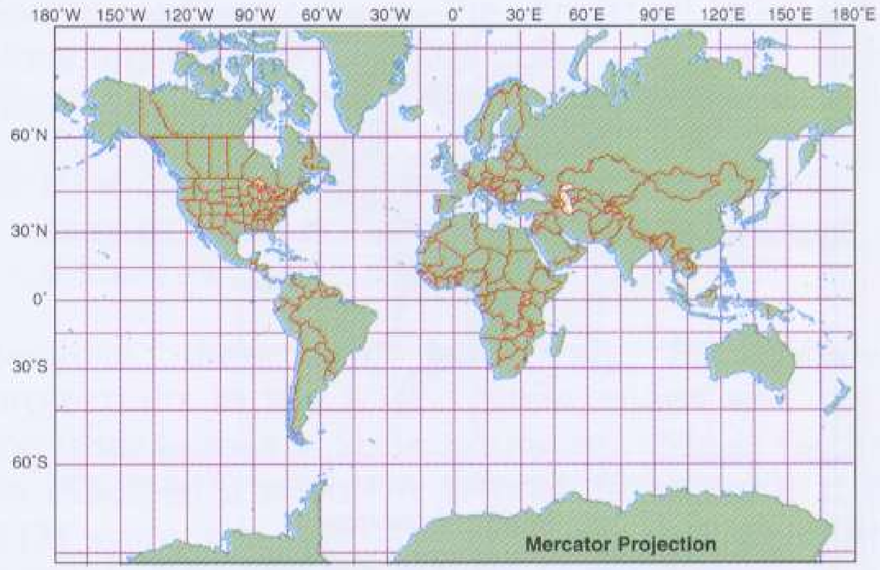
Cylindrical Projections

(Mercator)





Cylindrical Projection Surface



Universal Transverse Mercator (UTM) Coordinate System

- UTM system is transverse-secant cylindrical projection, dividing the surface of the Earth into 6 degree zones with a central meridian in the center of the zone.

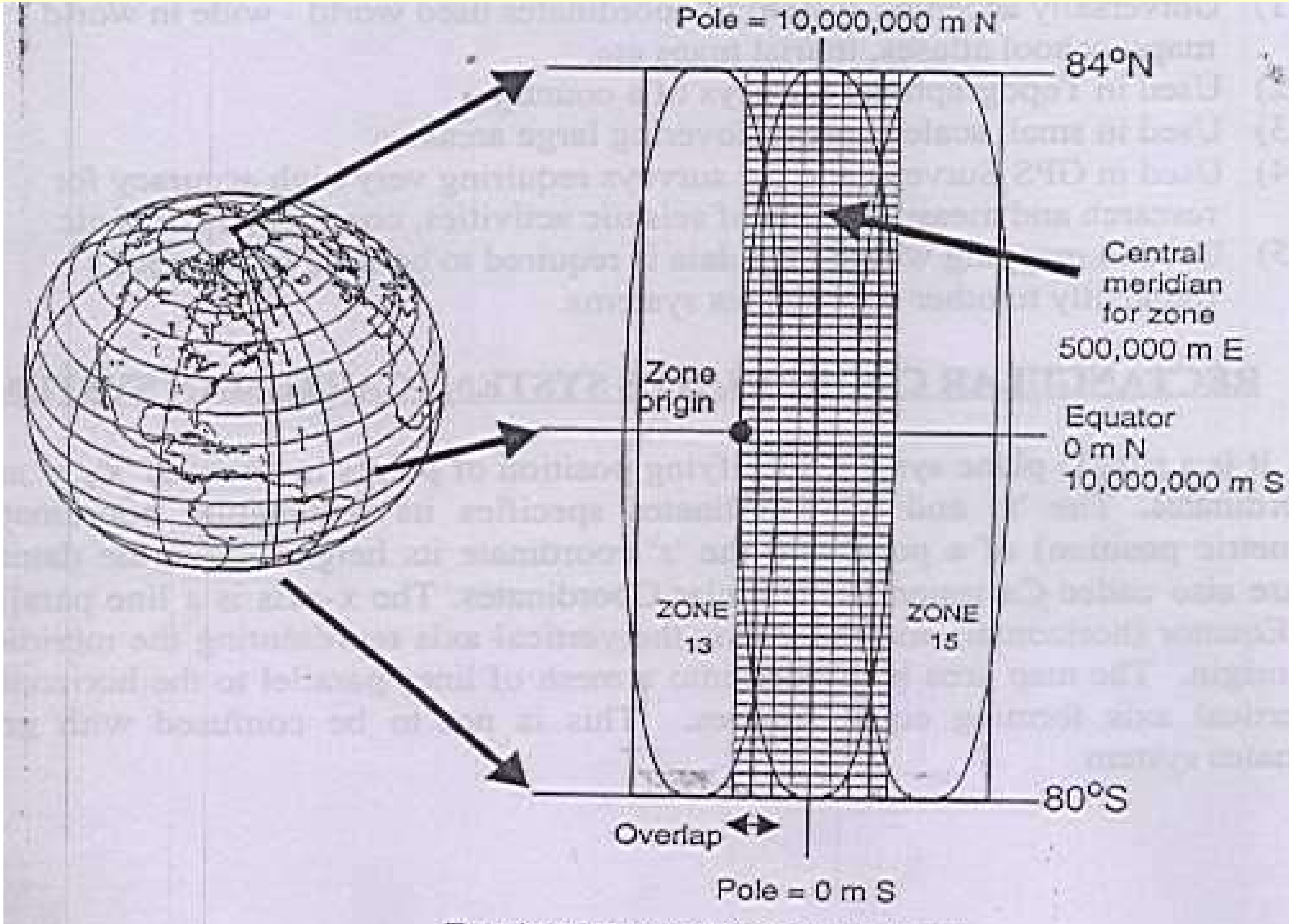
- Each zone is a different Transverse Mercator projection that is slightly rotated to use a different meridian. UTM zone numbers designate 6 degree longitudinal strips extending from 80 degrees South latitude to 84 degrees North latitude.

- UTM is a **conformal** projection, so small features appear with the correct shape. scale is same in all directions. (distances, directions, shapes, and areas are reasonably accurate). Scale factor is 0.9996 at the central meridian and at most 1.0004 at the edges of the zones.

- UTM coordinates are in meters, making it easy to make accurate calculations of short distances between points (error is less than 0.04%)

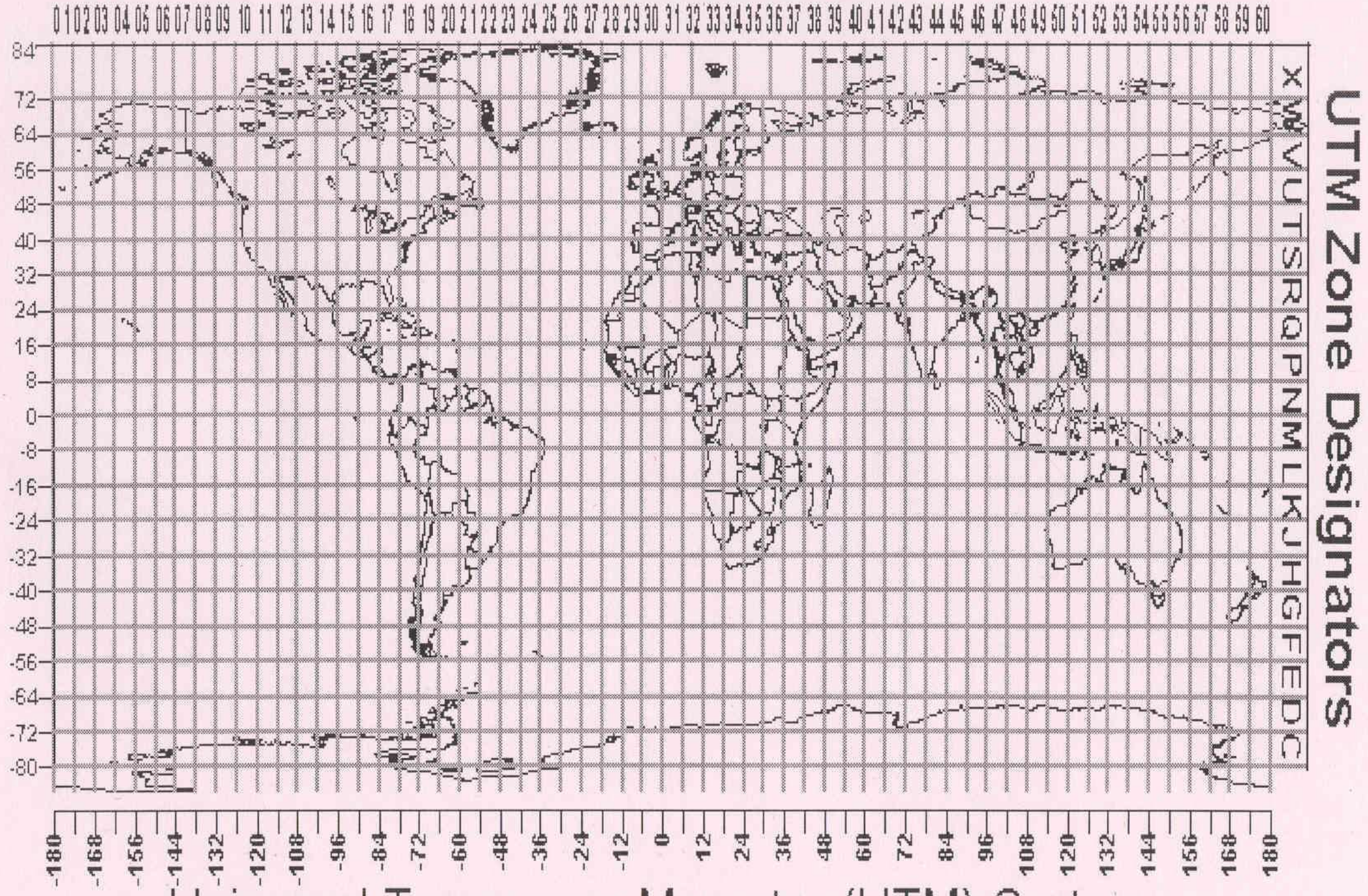
- Although the distortions of the UTM system are small, they are too great for some accurate surveying. zone boundaries are also a problem in many applications, because they follow arbitrary lines of longitude rather than boundaries between jurisdictions.

UTM PROJECTION



The universal transverse Mercator coordinate system.

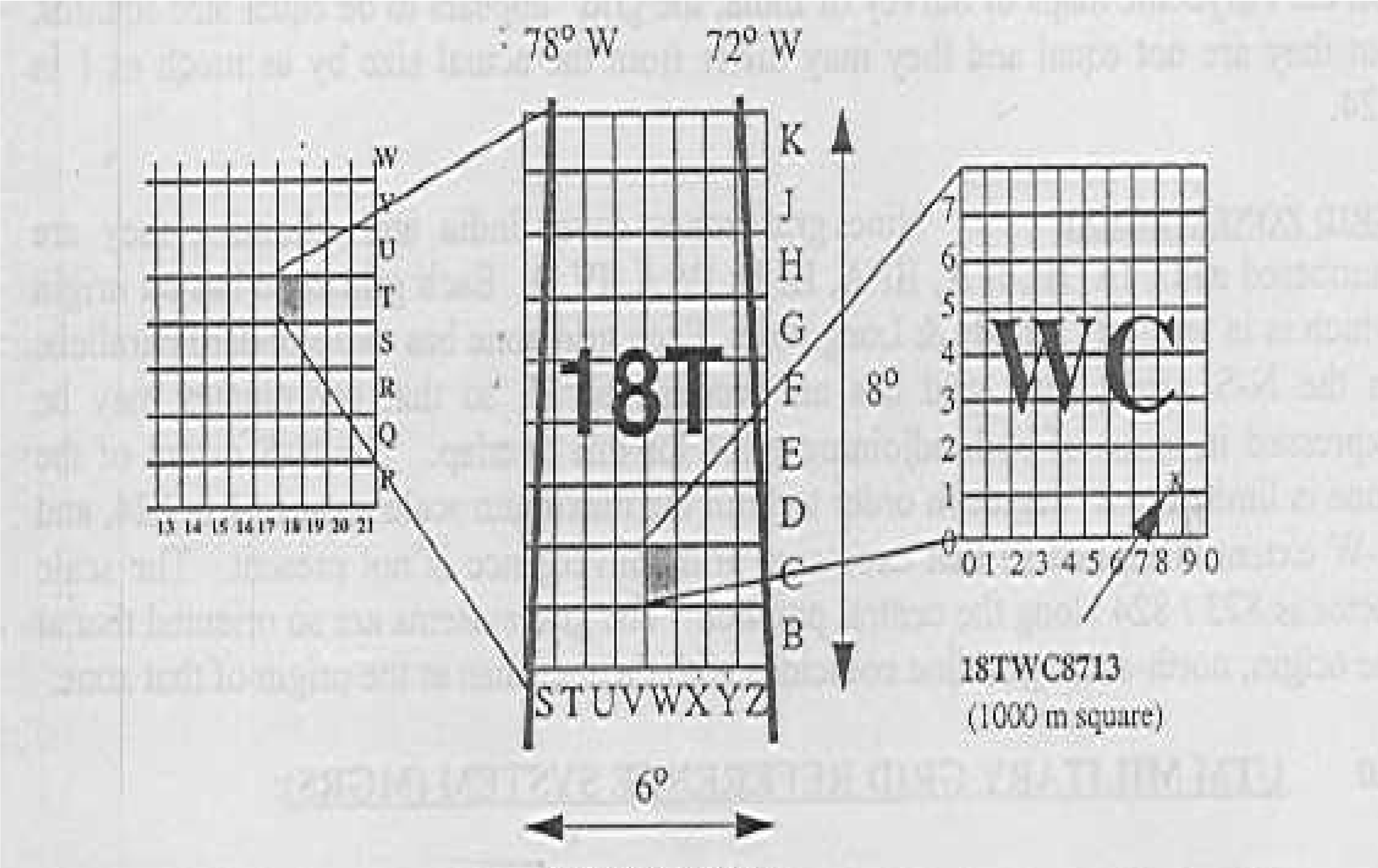
UTM Zone Numbers



Universal Transverse Mercator (UTM) System

Peter H. Dana 9/7/94

UTM PROJECTION

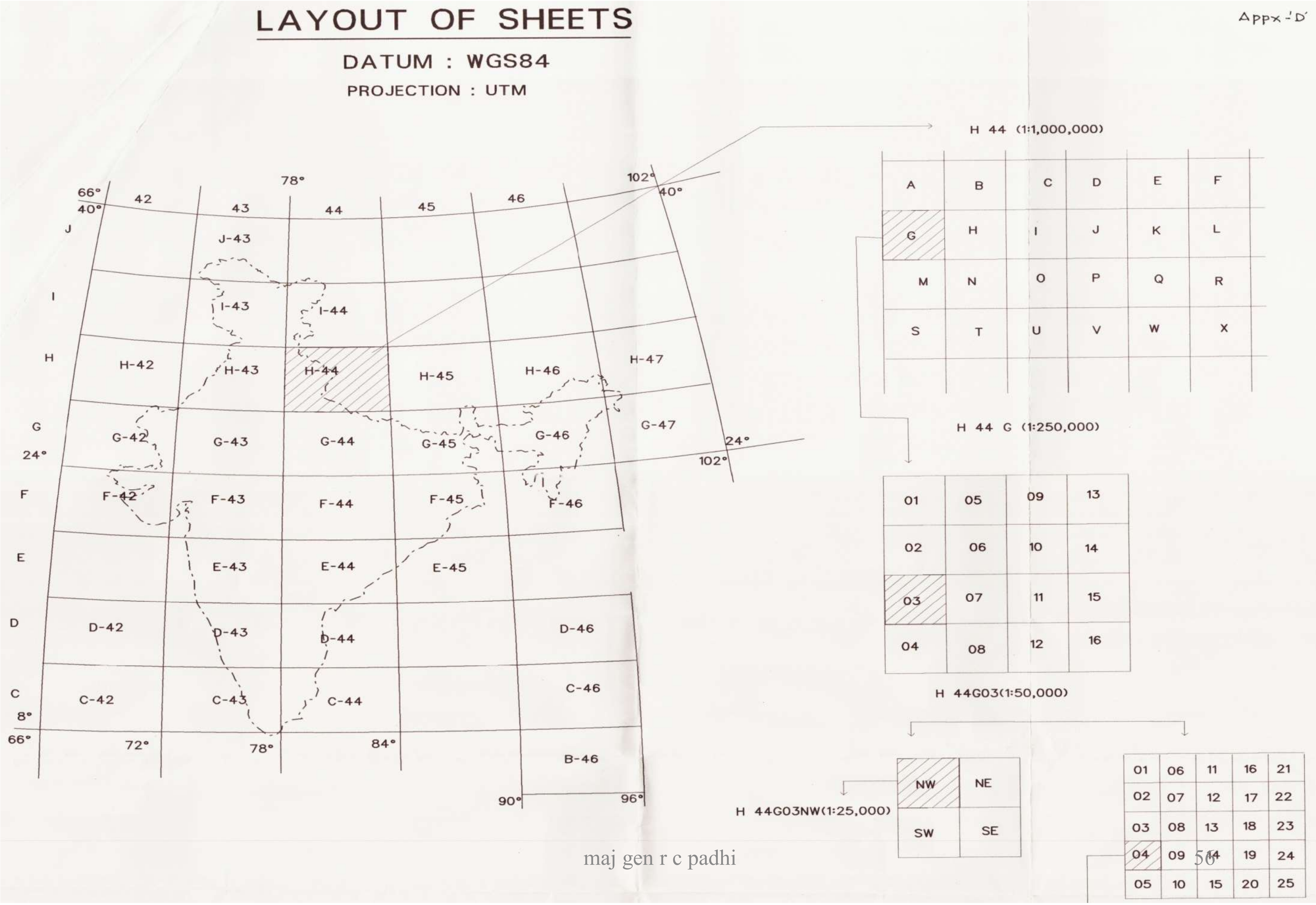


Military grid cell letters.

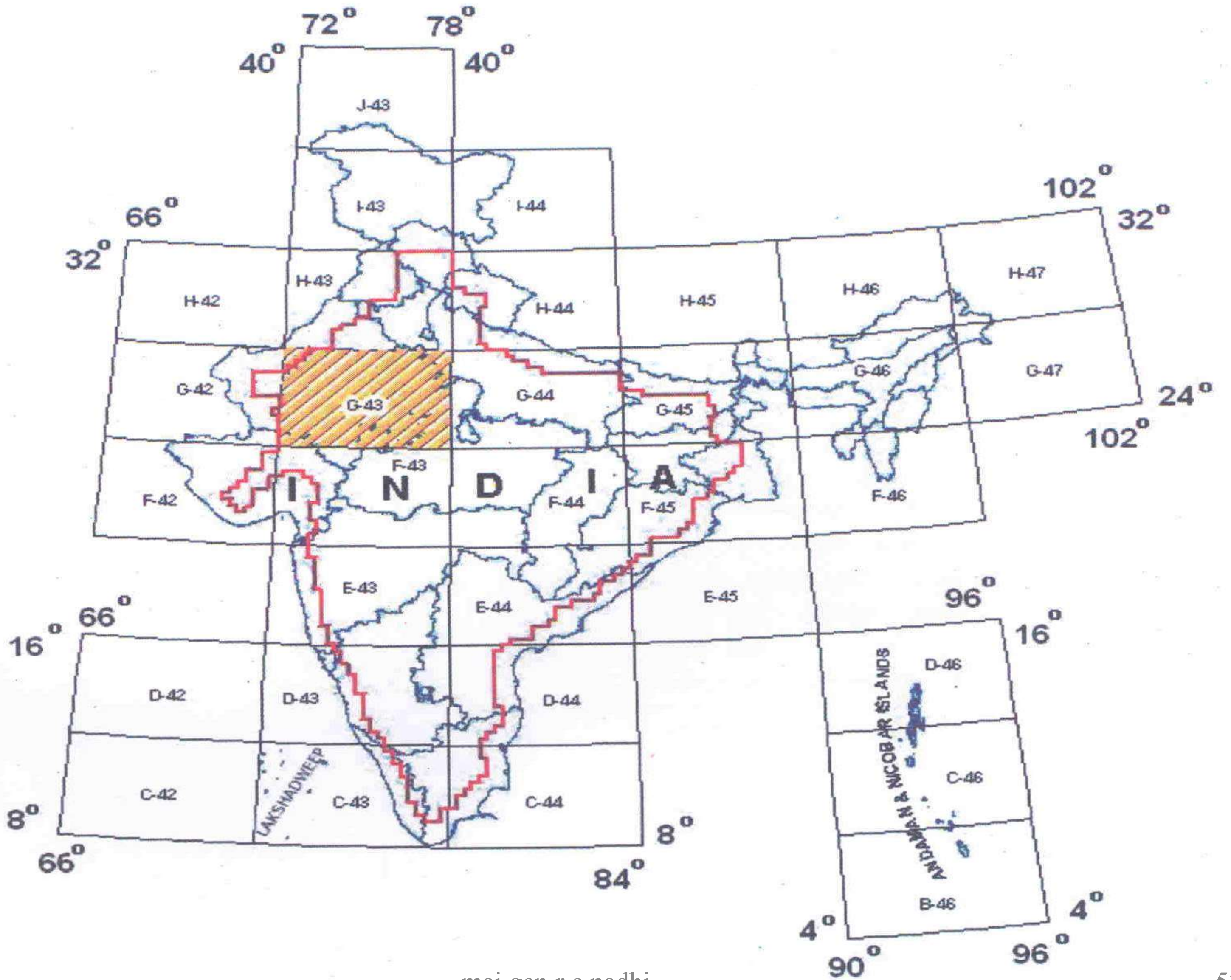
Universal Polar Stereographic (UPS) Coordinate System

- The UPS is defined above 84 degrees north latitude and south of 80 degrees south latitude.
- The eastings and northings are computed using a polar aspect stereographic projection.
- Zones are computed using a different character set for south and north Polar regions.

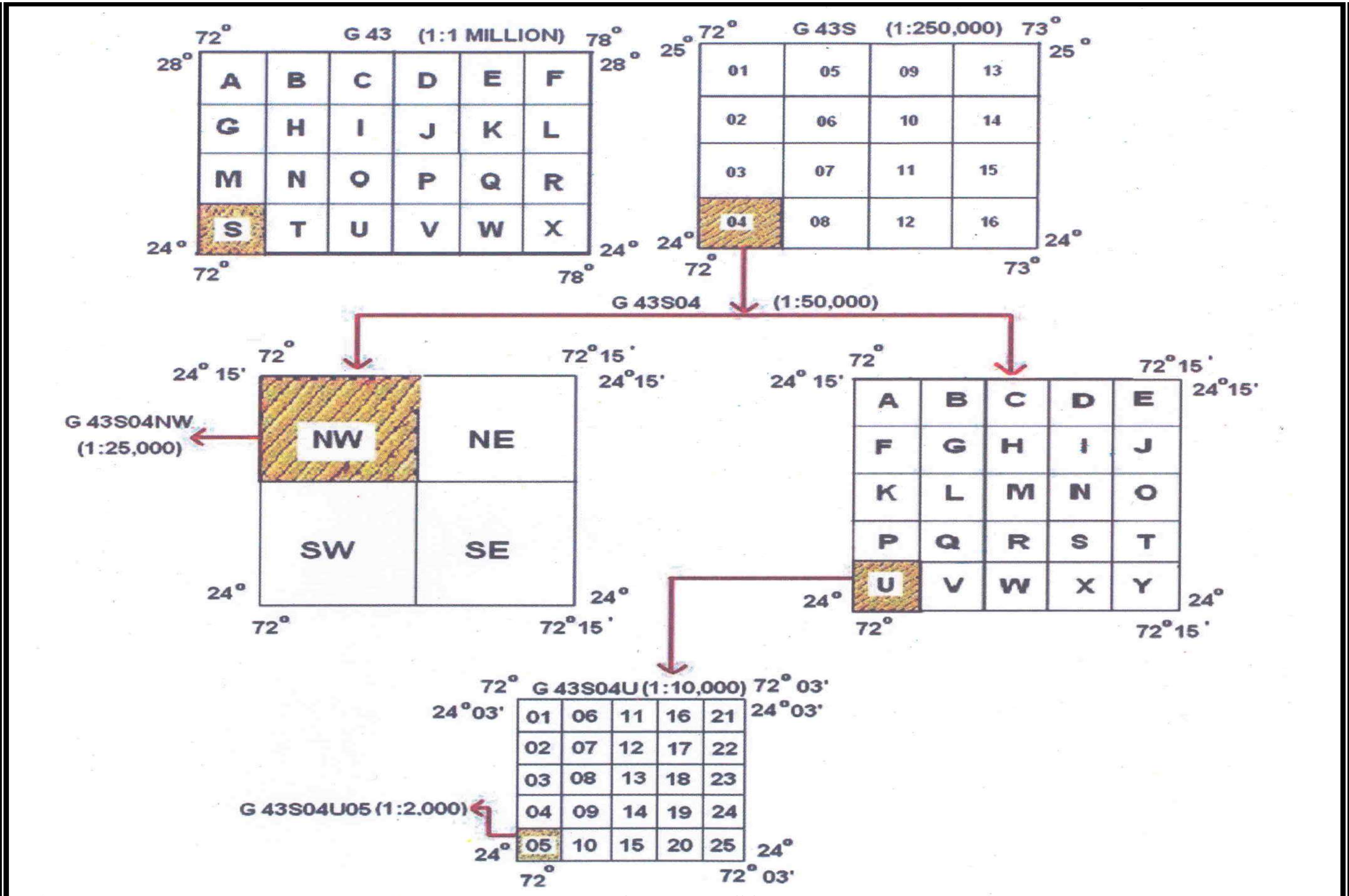
LAYOUT OF SHEETS



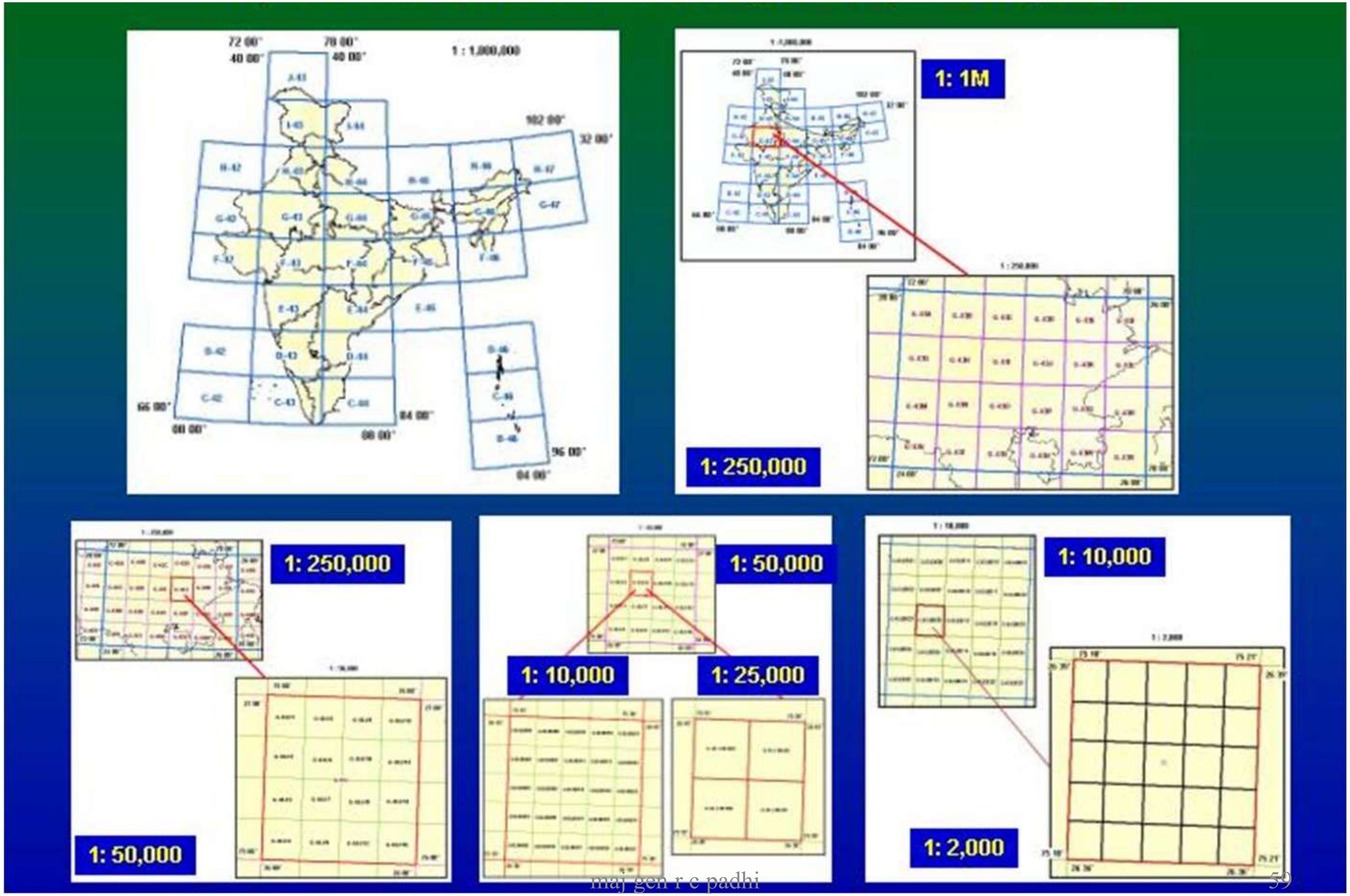
LAY OUT OF SHEETS



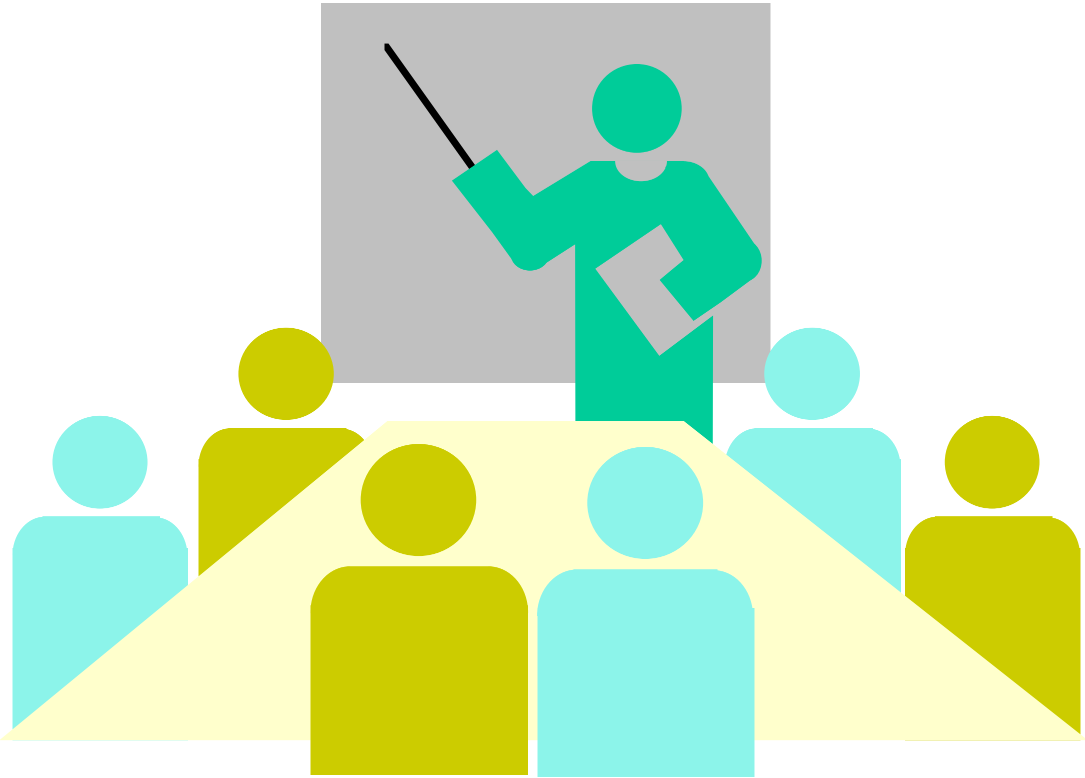
LAY OUT OF SHEETS



LAY OUT OF SHEETS
(as per national map policy)



Questions?



THANK YOU